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Original Origami

Introduction: This lesson provides a unique twist on creating origami. Instead of relying on traditional instructions and pictures, these origami instructions are written using mathematical terminology. They are tailored towards those who love creating origami without the ambiguity of unclear directions. Given the mathematical terminology used, these instructions are geared towards Geometry students.

NYS MST Standards:

- G.CM.11 Understand and use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and geometric diagrams.
- G.CM.12 Draw conclusions about mathematical ideas through decoding, comprehension, and interpretation of mathematical visuals, symbols, and technical writing.
- G.CN.7 Recognize and apply mathematical ideas to problem situations that develop outside of mathematics.
- G.G.38 Investigate, justify, and apply theorems about parallelograms involving their angles, sides, and diagonals.
- G.G.39 Investigate, justify, and apply theorems about special parallelograms (rectangles, rhombuses, squares) involving their angles, sides, and diagonals.
- G.G.57 Justify geometric relationships (perpendicularity, parallelism, congruence) using transformational techniques (translations, rotations, reflections).
- G.G.66 Find the midpoint of a line segment, given its endpoints.

NCTM Standards:

- Analyze properties and determine attributes of two- and three-dimensional objects.
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Instructional Objectives:

- Introduce an innovative way to create origami.
- Provide a link between mathematics and other disciplines, such as art.
- Use mathematical terminology as explicit instructions.

Instructional Protocol/Itinerary:

Students will be given a set of instructions and paper for two origami projects. Working alone or in teams of two, students will follow the directions to create their foldable artwork. Using the pictures and written instructions, students will see and understand the link between the mathematical terminology and the instructions for the creation. Students may not be familiar with the math terms used in the directions; however, using the diagrams and other descriptions, they should be able to figure out what the terms mean and develop their mathematical vocabulary. This lesson can be revised for any grade level.

A Truncated Pyramid

Required Materials:

- Pencil or pen
- 4"x4" paper

Assembly:

Step 1: Start with a square sheet of paper. Label the vertices of the square A, B, C and D . Fold side AB to side CD to form the horizontal line of symmetry EF . Note that E is the midpoint of side AD and F is the midpoint of side BC . Unfold to open the square.

Step 2: Fold vertex A to vertex C to form diagonal BD . Unfold to open the square.

Step 3: Fold vertex B to vertex D to form diagonal AC . All three folds will meet at the center G of the square. This will form three isosceles right triangles above and three isosceles right triangles below the horizontal bisector. (see Figure 1)

Step 4: With thumb and index finger, pinch the horizontal fold at midpoints E and F together towards the center to create an inside fold. This will form two isosceles right triangles, one in front of the other. E and F will meet at the midpoints of the hypotenuses of the right triangles. These points, together with G , determine the perpendicular bisectors of the hypotenuse. (see Figure 2)

Step 5: With vertex G on the bottom, fold vertex C across the triangle to meet the opposite side DG at point K and creating fold HI . The distance between point K and vertex G should be the same as the distance between vertex G and vertex I . (Point K is very close to, but just below, the midpoint of side DG .) (see Figure 3)

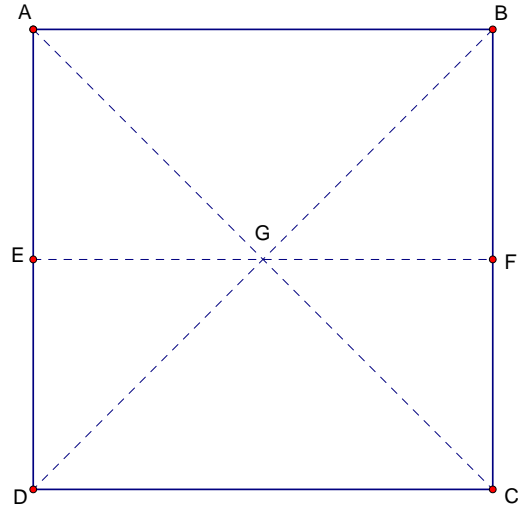


Figure 1

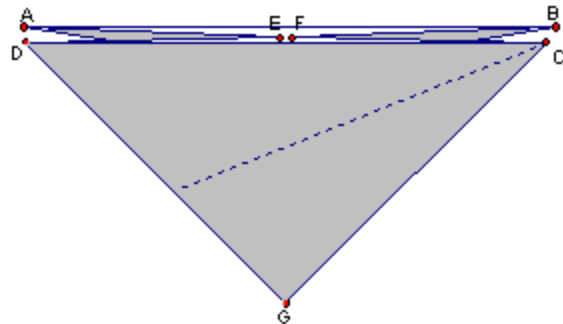


Figure 2

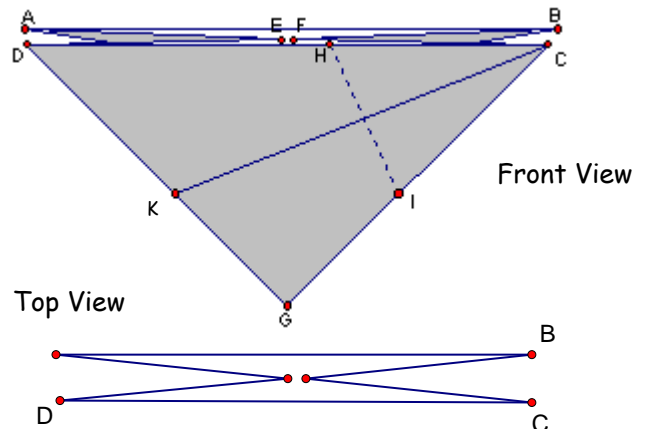


Figure 3

Step 6: Fold vertex D across the triangle to meet at the opposite side EG at I. This will form fold JK. Unfold and tuck vertex D inside triangle CHI. This will form isosceles trapezoid JHIK. (see Figures 4, 5 & 6).

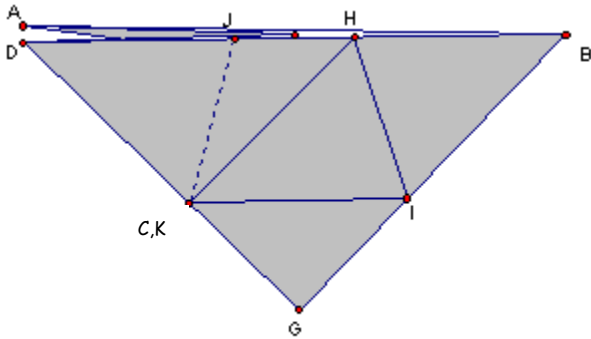


Figure 4

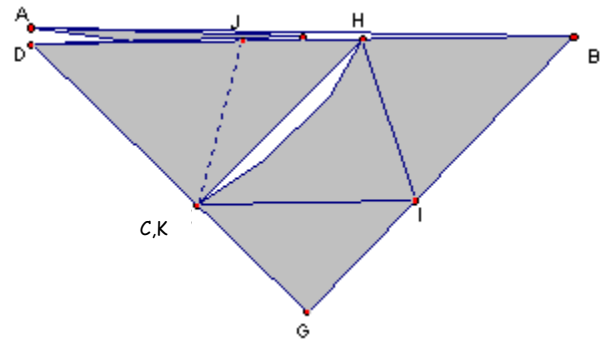


Figure 5

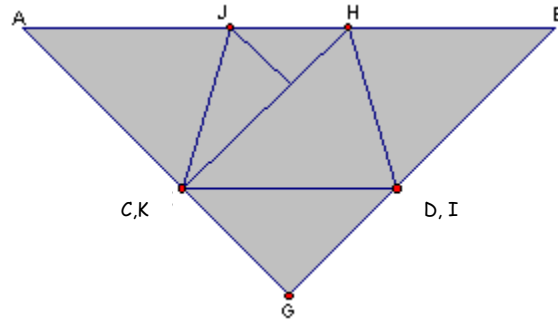


Figure 6

Step 7: Flip the paper over so the backside is now on the front with vertex G on bottom. Fold vertex A across the triangle to side BG, creating crease LM; A should meet side BG at point O such that $GO=GI$. Also, GM should equal GK . (see Figure 7)

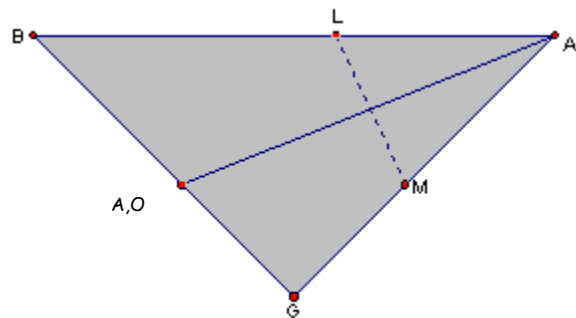


Figure 7

Step 8: Fold vertex B across the paper to meet the opposite side AG at M. This will create fold NO. Unfold and tuck inside triangle ALM. The result is an irregular pentagon. (see Figures 8 & 9)

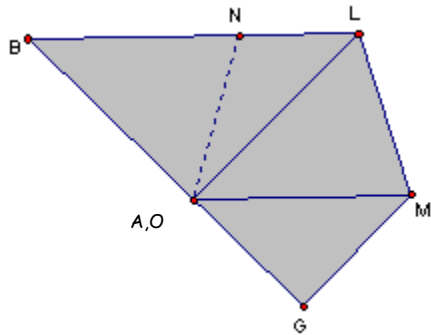


Figure 8

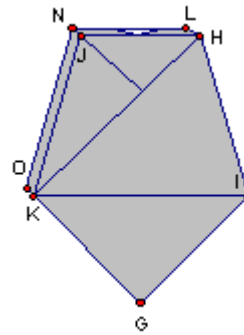


Figure 9

Step 9: Pull open the top of the pentagon, and push up the bottom to create squares HLNJ and IMOK. This will create a truncated square pyramid; point G will be the center of the base. (see Figure 10)

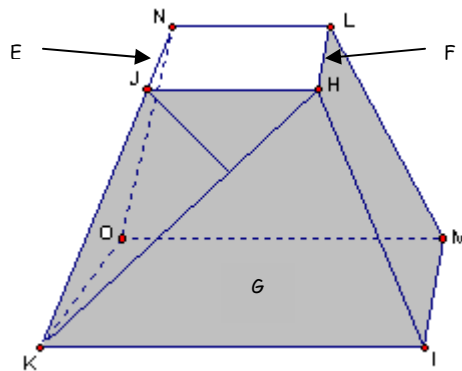


Figure 10

Note: When students are finished with their truncated pyramid, give them a fun treat to place in their pyramids, as it will be big enough to hold small objects. Examples of treats that can be used are M&Ms, jelly beans or candy corn.

As students finish their projects, allow them to help other students complete their projects. As an additional assignment, you may have students re-write the directions using their own mathematical terminology or terms currently used in the course.

Stellated Octahedron

This origami project involves two distinct processes: the manufacture of component pieces and the assembly of the octahedron.

Required Materials:

- Twelve 4"x4" square pieces of paper per person or team
- A pen or pencil

Assembly of Component Pieces:

1. Start with a 4"x4" square; label it ABCD as shown in Figure 1.

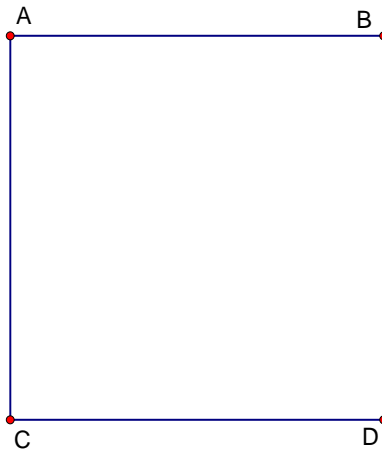


Figure 1

2. Fold \overline{AB} onto \overline{CD} , the crease created will be the horizontal axis of symmetry of the square. After you have made the crease, unfold and label the end points E and F. E will be the midpoint of \overline{AC} and F will be the midpoint of \overline{BD} . (See Figure 2).

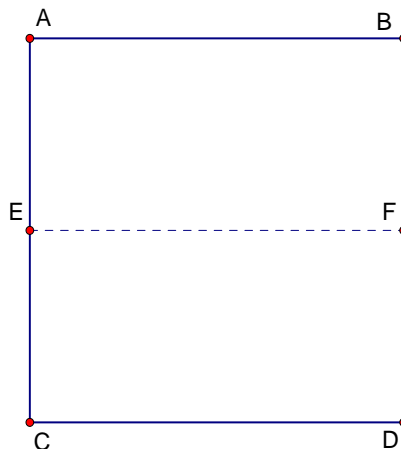


Figure 2

3. Fold \overline{AB} onto \overline{EF} and unfold. Label G (the midpoint of \overline{AE}) and I (the midpoint of \overline{BF}). Next, fold \overline{CD} onto \overline{EF} and unfold. Label J (the midpoint of \overline{EC}) and K (the midpoint of \overline{FD}). You should now see four congruent rectangles: $ABIG$, $GIFE$, $EFKJ$, and $JKDC$. (See Figure 3.)

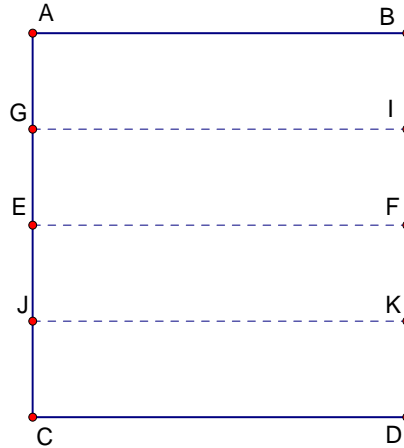


Figure 3

4. Fold vertex A onto \overline{GI} making an isosceles right triangle. Label it $\triangle GAH$. Keep this triangle folded. Fold vertex D onto \overline{KJ} making an isosceles right triangle. Label it $\triangle KDL$. Again, keep this triangle folded. (See Figure 4.)

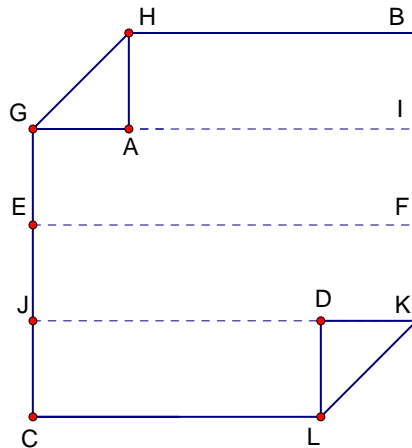


Figure 4

5. Fold \overline{GH} so it lies along \overline{GI} , forming obtuse isosceles triangle $\triangle GHP$. Keep this triangle folded. Fold \overline{KL} so it lies along \overline{JK} , forming obtuse isosceles triangle $\triangle KLQ$. Again, keep this triangle folded. (See Figure 5.)

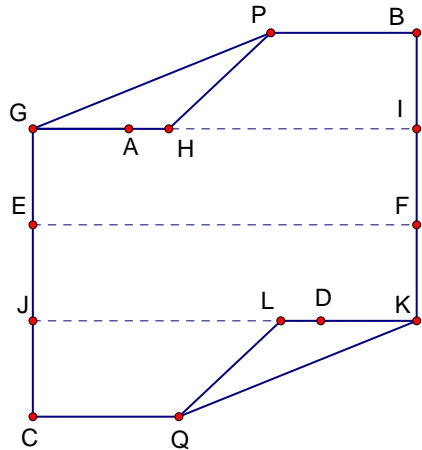


Figure 5

6. Fold \overline{PB} onto \overline{EF} so that B meets F ; similarly fold \overline{CQ} onto \overline{EF} so that C meets E . You should now have a rectangle with vertices $G, I, K,$ and J . (See Figure 6.)

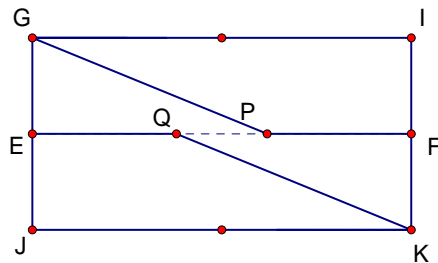


Figure 6

7. Fold \overline{GJ} onto \overline{IK} , and then unfold, creating crease \overline{TU} , where T is the midpoint of \overline{GI} and U is the midpoint of \overline{JK} . Next, fold vertex J to midpoint T and vertex I to midpoint U , creating fold \overline{GU} and \overline{KT} . Unfold and then fold vertex J underneath \overline{GP} and fold vertex I under \overline{QK} . (See Figure 7.)

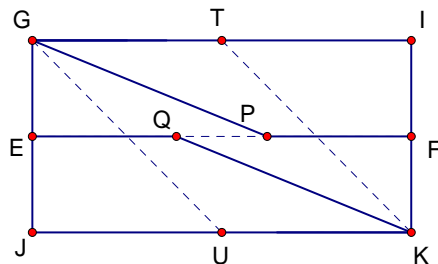


Figure 7

8. We now have a parallelogram, $\square GTKU$. Reflect about \overline{TU} , and then rotate so vertex G is on the top right corner, as oriented in the diagram. (See Figure 8.)

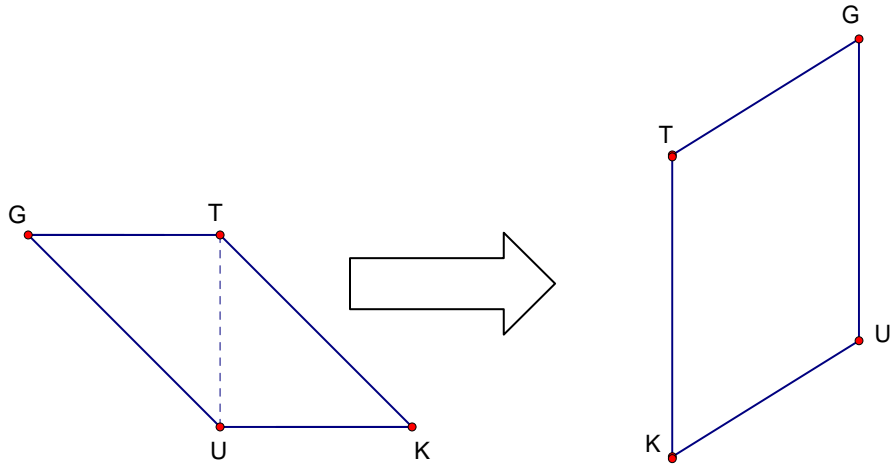


Figure 8

9. Fold vertex K to vertex T and fold vertex G to vertex U . This forms square $KZGY$, where Z was the midpoint of \overline{GU} and Y was the midpoint of \overline{KT} before these two folds were made. Fold vertex Y to vertex Z and unfold, creating diagonal crease from K to G . (See Figure 9.)

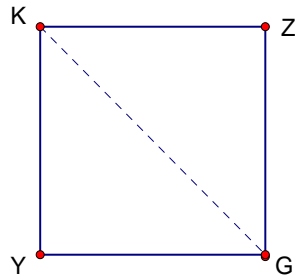


Figure 9

10. Unfold vertex Y from vertex Z and unfold vertex K and vertex G . This shape is now ready for assembly, as shown in the Figure 10. Our shape should look like four isosceles right triangles when unfolded.

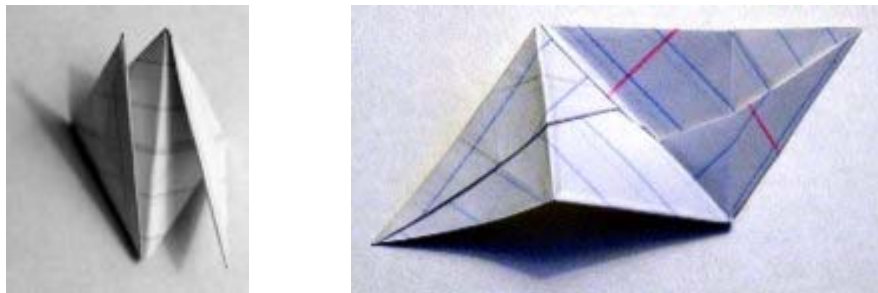


Figure 10

11. Repeat steps 1 - 10 for the remaining 4"x4" squares.

Assembly of the Stellated Octahedron:

1. After completing the assembly of all 12 component pieces, we will number the points and pockets.
2. Start with the top point of the parallelogram, and label that point 1. Following the diagonal along the parallelogram from top to bottom, label points 2, 3 and 4 (See Figure 11 for additional help.)

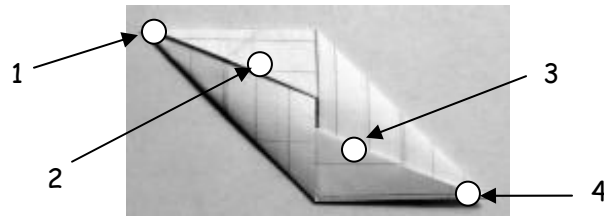


Figure 11

3. Repeat step 2 with the next piece. However, in this piece the top point will be labeled 5; continue with 6, 7, and 8.
4. Repeat with the remaining 10 pieces, number in ascending order following step 2 (Figure 11) each time, and ending with number 48 (See Figure 12).

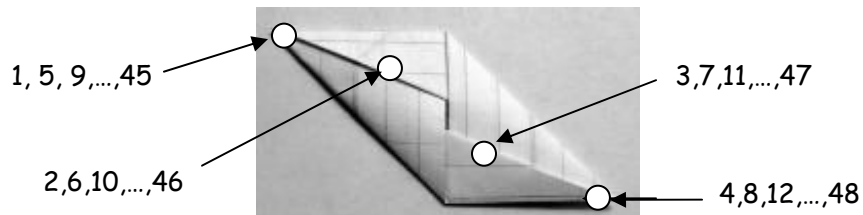
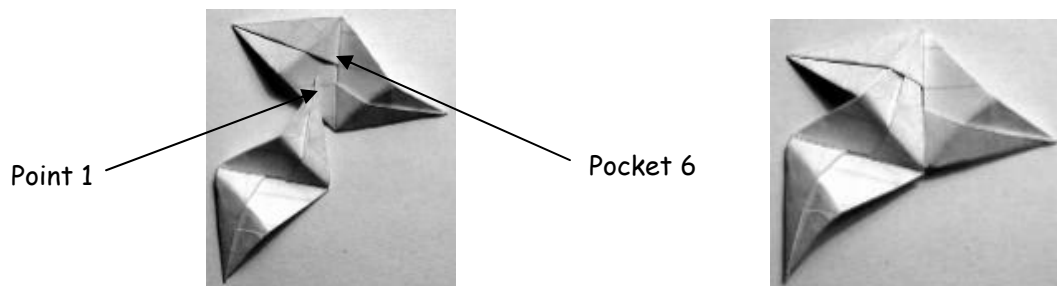


Figure 12

5. Now, follow the chart to assemble the octahedron. (ex. STEP 1: POINT 1 goes into POCKET 6)



Stellated Octahedron Assembly Chart

STEP	POINT	POCKET
1	1	6
2	5	10
3	9	2
4	4	14
5	13	18
6	17	3
7	8	22
8	24	19
9	12	26
10	16	30
11	32	11
12	25	31
13	20	34
14	33	23
15	21	38
16	37	7
17	28	39
18	29	42
19	41	15
20	36	43
21	40	46
22	48	35
23	44	47
24	45	27



This is what you will see when three pieces are assembled (Steps 1-3).

Completed Origami Stellated Octahedron



Additional pictures and more complicated similar projects can be found on this web page:
<http://nuwen.net/poly.html>

Other projects using the same component pieces include:

- Cube (4 pieces)
- Icosahedron (30 pieces)
- Stellated truncated icosahedron (270 pieces)