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Pluck my String!

Introduction:

Every time you listen to music you are listening to mathematical ratios. These mathematical ratios determine the sounds that are made by a plucked string. Using our knowledge of ratios we can determine the multiple sounds that a single string will make when plucked and relate this back to our knowledge of music.

NYS Standards:

- G.CM.1 Communicate verbally and in writing a correct, complete, coherent, and clear design (outline) and explanation for the steps used in solving a problem.
- G.CN.6 Recognize and apply mathematics to situations in the outside world.
- G.CN.8 Develop an appreciation for the historical development of mathematics.
- A.RP.2 Use mathematical strategies to reach a conclusion and provide supportive arguments for a conjecture.

NCTM Standards:

- Recognize and apply mathematics in contexts outside of mathematics.
- Solve problems that arise in mathematics and in other contexts.
- Understand and use ratios and proportions to represent quantitative relationships.
- Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers.

Instructional Objectives:

* Students will be able to calculate ratios.
* Use these ratios to create the different string lengths.
* Relate these patterns to proportions and music octaves.
* Use what they have found mathematically in music to relate to real world usage.
* Use what they have learned from the proportions used by Pythagoras to find the different notes in the modern major scale.
* Students will relate proportions to other situations outside the classroom.

Instructional Protocol/Itinerary:

* Give the students a history on Pythagoras and how Pythagoras used these proportions.
* Using ratios, students will use this knowledge to construct different string lengths to produce different musical notes and create an F Lydian scale.
* The student will use a piece of string and by using different tensions and lengths deduce different properties of string length and tension.
* This lesson can be revised to make appropriate for different grade levels and extended to more intricate mathematics.

Materials needed for this lesson:

String, rubber bands, jump ropes, ruler, pencil, paper, and calculator

 Music of the Spheres 

The first person to make the connection between math and music was Pythagoras of Samos, a famous philosopher and cult leader who lived most of his life in southern Italy in 5th century BC. Not only the creator of the oldest known proof of the ____________, but also looked at the connections between music and mathematics. For Pythagoras, ratios were everything. He believed every value could be expressed as a ___________, it can also be thought of as a rational number (he was wrong, but that is a whole different story). He also is the first to believe in the idea that mathematics was everywhere.

Definitions:
Octave-
Major Scale-
Mode-
Intervals-

In pairs:
1) Measure the length of the string and record the length below.

2) Hold the string tight and pluck it.

3) Measure half the distance of the string and hold at this point. Then hold the string at the same tightness and pluck it. State your observations below.

4) Split the string into a fourth of the original length and record your observations.

5) Repeat the process with different lengths and tightness. Record your observations.

In ancient Greece, music was not as complicated as it is today. The Greek octave was built upon five notes. Pythagoras deduced that each note in the Greek scale was a fraction of a string.

Sounding the Ratios

Musical scales have become more complicated as time has progressed. It has changed from a five-note scale to an eight-note major scale. How was this constructed?

Using a mode of the C major scale, also known as the F Lydian scale, we can calculate the ratios that will yield the notes we are looking for. The F Lydian mode is the C major scale played from F to an F an octave above.

We have found that half the length of a string produces the same note at a higher pitch, or an octave higher. This ratio is 1:2. Where the longer string is twice the length of the shorter string. Pythagoras found that a fifth is the ratio of 3:2, which the longer string is three times the size of twice the shorter string. Answer the following questions in the construction of the F Lydian mode.

1) Start with note F, which has a ratio of 1:1. Find the ratio needed for the string to sound note C, which is the fifth above F.

2) Take a fifth above C, which is G, what is this ratio? This ratio lies outside our octave, how can we move this note into our octave?

3) Take a fifth above G, which is D, what is the ratio?

4) Take a fifth above D, which is A, what is the ratio? Hint: this lies outside our octave.

5) Take a fifth above A, which is E, what is the ratio?

6) Take a fifth above E, which is B, what is the ratio? Hint: this lies outside our octave.

f-g-a-b-c-d-e-f

We have found our modern 8-note mode of the C major scale, using all natural notes. This is just a single scale from our modern western musical system. This is also the only scale to use all natural notes. All other major scales in modern music use sharps and flats! With further analysis, one can calculate the ratios for sharps and flats using similar techniques!

Build Me a Harp!

Shopping at the local music store, the owner will explain the guitar in a peculiar fashion. He will begin with the different types of wood that are used to create a guitar. The owner will then explain the fret board. He will state, "The only thing that separates a real guitar from a toy guitar is that a mathematical formula is used to calculate the precise distances of the frets." Thus, math is what makes a guitar a guitar. But, the guitar is not the only instrument that is created from math. As we have found, all stringed instruments are created by math. A harp is one of the best examples of these ratios in use. With our knowledge of ratios, can you build me a harp?

Note: Every string on the harp will be of the same tightness. We are also finding the length of the short string, thus adjust the ratio appropriately to find the length of the shorter string. Remember these ratios are long string to short string!

1) The lowest string on the harp is going to be 2.5 feet. This string will be tuned to f just as our Lydian mode we created. Calculate the string length in inches.

2) Find the string length of the string sounding an octave above the low string.

3) Find the string length that will sound the note g of our F Lydian scale.

4) Continue to calculate the all of the string lengths, these will be the lengths for the notes, a, b, c, d, and e. Place your calculations in the table below.

Note	Ratio	String length (inches)
f	1:1	
g		
a		
b		
c		
d		
e		
f		

5) Draw a picture below of the harp and label the strings with their string lengths.

ANSWER SHEET:

Music of the Spheres:

Fill in the blank:

1) Pythagorean Theorem
2) Fraction

Definitions:
Octave- An octave is one of the 12 primary notes but at a higher or lower frequency. i.e. a note that is an octave higher than a reference note is at twice the frequency of the original note.

Major Scale- In music theory, the major scale is one of the diatonic scales. It is made up of seven distinct notes, plus an eighth, which duplicates the first, an octave higher.

Mode- Any of various fixed orders of the various diatonic notes within an octave

Intervals- Intervals are musical distances between notes. When a note is a 5th from another, one will count five notes including the beginning and ending note. For example a 5th above c would be g, c-d-e-f-g, a distance of five notes.

1) Answers will vary depending on string length

3) Students should find that the string at half-length sounds the same as the string at full length, but at a higher pitch.

4) A fourth will also create the same sound as the original, but at a higher pitch.

5) Students should find that using half the length or a fourth of the length does not change the note played, but changes its pitch. Students should also deduce that different tightness changes the note played by the string.

Sounding the Ratios:

1) 3:2
2) (3:2)*(3:2)=9:4
Since it lies outside our octave, multiply 9:4 by 1:2 to get 9:8 for note D.
3) (9:8)*(3:2)= 27:16
4) (27:16)*(3:2)=81:32
Lies outside our octave: (81:32)*(1:2)=81:64
5) (81:64)*(3:2)=243:128
6) (243:128)*(3:2)=729:256
Lies outside our octave: (729:256)*(1:2)=729:512

Build Me a Harp!

1) $2\frac{1}{2}\,feet \times \frac{12 inches}{1 foot} = 30 inches$

2) The ratio for the string an octave above our original string is 1:2. Thus the short string length is,

$$30 inches \times \frac{1}{2} = 15 inches$$

3) For the string to sound the g note, we must use the ratio 9:8, but this is long string to short string. Thus to find the length of the short string,

$$30 inches \times \frac{8}{9} = 26.67 inches$$

4) Using the same procedure,

Note	Ratio	String Length (inches)
f	1:1	30
g	9:8	26.67
a	81:64	23.7
b	729:512	21.07
c	3:2	20
d	27:16	17.78
e	243:128	15.8
f	1:2	15

5) Answers will vary, but the picture should show each consecutive string becoming smaller as the notes sound from the low f note to the higher f note.