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Check Mate-rix

This lesson is to provide students with hands on learning technique to help solve matrices using the Gauss and Gauss-Jordan Elimination for pre-calculus and calculus students. This should be used as a "day 2" activity to reinforce the method of solving for the reduced row echelon form of matrices.

Standards:

This lesson addresses the following New York State Standards:

- A2.CM.5: Communicate logical arguments clearly, showing why a result makes sense and why the reasoning is valid.
- A2.CM.11: Represent word problems using standard mathematical notation.
- G.PS.9: Interpret solutions within the given constraints of a problem.
- A2.CN.1: Understand and make connections among multiple representations of the same mathematical data.
- A2.R.1: Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects created using technology as representations of mathematical concepts.
- A2.R.3: Use representation as a tool for exploring and understanding mathematical ideas.

Objectives:

Upon completion of this lesson, students will be able to do the following:

- Understand the rules to manipulate matrices.
- Visually and physically be able to solve matrices in the reduced row echelon form.
- Be able to identify word problems with systems of linear equations containing more than one unknown variable and be able to solve them through the use of matrices.

Instructional Protocol:

First have the students try the "hook" exercise on their own. The majority of the students should be using substitution to solve a system of equations with two unknowns. Moving on to the next problem with three equations and three unknowns, substitution becomes more difficult and the teacher should introduce using matrices to solve the problem. The teacher should ask the students how they would solve it using the new skills learned about the Gauss-Jordan elimination method. Introduce the checkerboard method with the hook problem. This is clearly more time consuming than substitution. However, have the students work in small groups or pairs to solve the larger checkerboard matrices. Competition is always appreciated in these sorts of activities. You may want to have a race against the groups and yourself, or pit the students against one another. The checkerboard game is great for visual learners.

Quick Refresher of Gauss-Jordan Elimination:

$$I_1 = \begin{bmatrix}1\end{bmatrix},\ I_2 = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix},\ I_3 = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix},\ \cdots,\ I_n = \begin{bmatrix}1 & 0 & \cdots & 0\\0 & 1 & \cdots & 0\\\vdots & \vdots & \ddots & \vdots\\0 & 0 & \cdots & 1\end{bmatrix}$$

These are examples of the Identity Matrices. They are the final result after Gauss-Jordan Elimination.

Gauss-Jordan elimination brings a matrix to reduced row echelon form to solve systems of equations. By performing elementary row operations on the matrices, we can achieve the Identity Matrix. These elementary row operations are switching rows, multiplying a row by constant, or adding (or subtracting) rows from one another which preserve matrix equivalence.

Let's go to the movies:

Matrix Evolution

There is a new spoof coming out this Friday, titled "Matrix Evolutions", and of course we are very excited to see it and enjoy some tasty treats. We will be going in 2 groups, the freshmen, and the upper classmen. There are 3 freshmen. They each buy popcorn and their ticket and their total all together is $24. The group of upper classmen consists of 2 couples. Each boyfriend buys their girlfriend popcorn to share together on their date. The second group's total is $26 including their four tickets. What was the cost of each movie ticket and each popcorn?

Solution: (2 methods)

3x+3y=24
4x+2y=26

Method 1: Substitution!!

$$3x + 3(y) = 24$$
$$y = 13 - 2x$$
$$3x + 3(13 - 2x) = 24$$
$$3x + 39 - 6x = 24$$
$$3x = 15$$
$$\boldsymbol{x = 5}$$
$$y = 13 - 2(5)$$
$$\boldsymbol{y = 3}$$

Method 2: Matrices and Gauss-Jordan Elimination!!

$$\begin{bmatrix} 3 & 3 & 24 \\ 4 & 2 & 26 \end{bmatrix} \xrightarrow{R_2 - R_1 \to R_1} \begin{bmatrix} 1 & -1 & 2 \\ 4 & 2 & 26 \end{bmatrix}$$

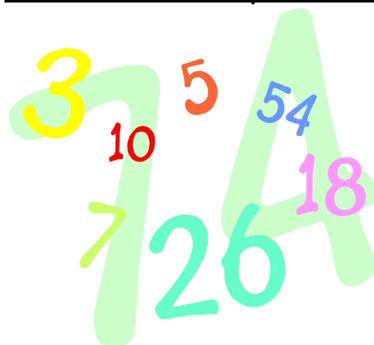
$$\begin{bmatrix} 1 & -1 & 2 \\ 4 & 2 & 26 \end{bmatrix} \xrightarrow{-4R_1 + R_2 \to R_2} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 6 & 18 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 6 & 18 \end{bmatrix} \xrightarrow{\frac{1}{6}R_2 \to R_2} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 3 \end{bmatrix} \xrightarrow{R_2 + R_1 \to R_1} \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 3 \end{bmatrix}$$

Therefore the movie tickets are $5 and each popcorn is $3.

<u>**Pick a Number, Any Number!**</u>



Let's use matrices to solve this problem:

The sum of 3 numbers is 38. The 3rd number is 4 times the 2nd number and we know that 3 times the 1st number and 3 times the 2nd number is 30.

Solution:

$$a + b + c = 38$$
$$4b - c = 0$$
$$3a + 3b = 30$$

$$\begin{bmatrix} a & b & c & 38 \\ 0 & 4b & -c & 0 \\ 3a & 3b & 0 & 30 \end{bmatrix} \xrightarrow{yields} \begin{bmatrix} 1 & 1 & 1 & 38 \\ 0 & 4 & -1 & 0 \\ 3 & 3 & 0 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 38 \\ 0 & 4 & -1 & 0 \\ 3 & 3 & 0 & 30 \end{bmatrix} \xrightarrow{-3R_1+R_3 \to R_1} \begin{bmatrix} 1 & 1 & 1 & 38 \\ 0 & 4 & -1 & 0 \\ 0 & 0 & -3 & -84 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 38 \\ 0 & 4 & -1 & 0 \\ 0 & 0 & -3 & 84 \end{bmatrix} \xrightarrow{-\frac{1}{3}R_3 \to R_3} \begin{bmatrix} 1 & 1 & 1 & 38 \\ 0 & 4 & -1 & 0 \\ 0 & 0 & 1 & 28 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 38 \\ 0 & 4 & -1 & 0 \\ 0 & 0 & 1 & 28 \end{bmatrix} \xrightarrow{-\frac{1}{4}R_2 \to R_2} \begin{bmatrix} 1 & 1 & 1 & 38 \\ 0 & 1 & \frac{-1}{4} & 0 \\ 0 & 0 & 1 & 28 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 38 \\ 0 & 1 & \frac{-1}{4} & 0 \\ 0 & 0 & 1 & 28 \end{bmatrix} \xrightarrow{\frac{1}{4}R_3+R_2 \to R_2} \begin{bmatrix} 1 & 1 & 1 & 38 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 28 \end{bmatrix}$$

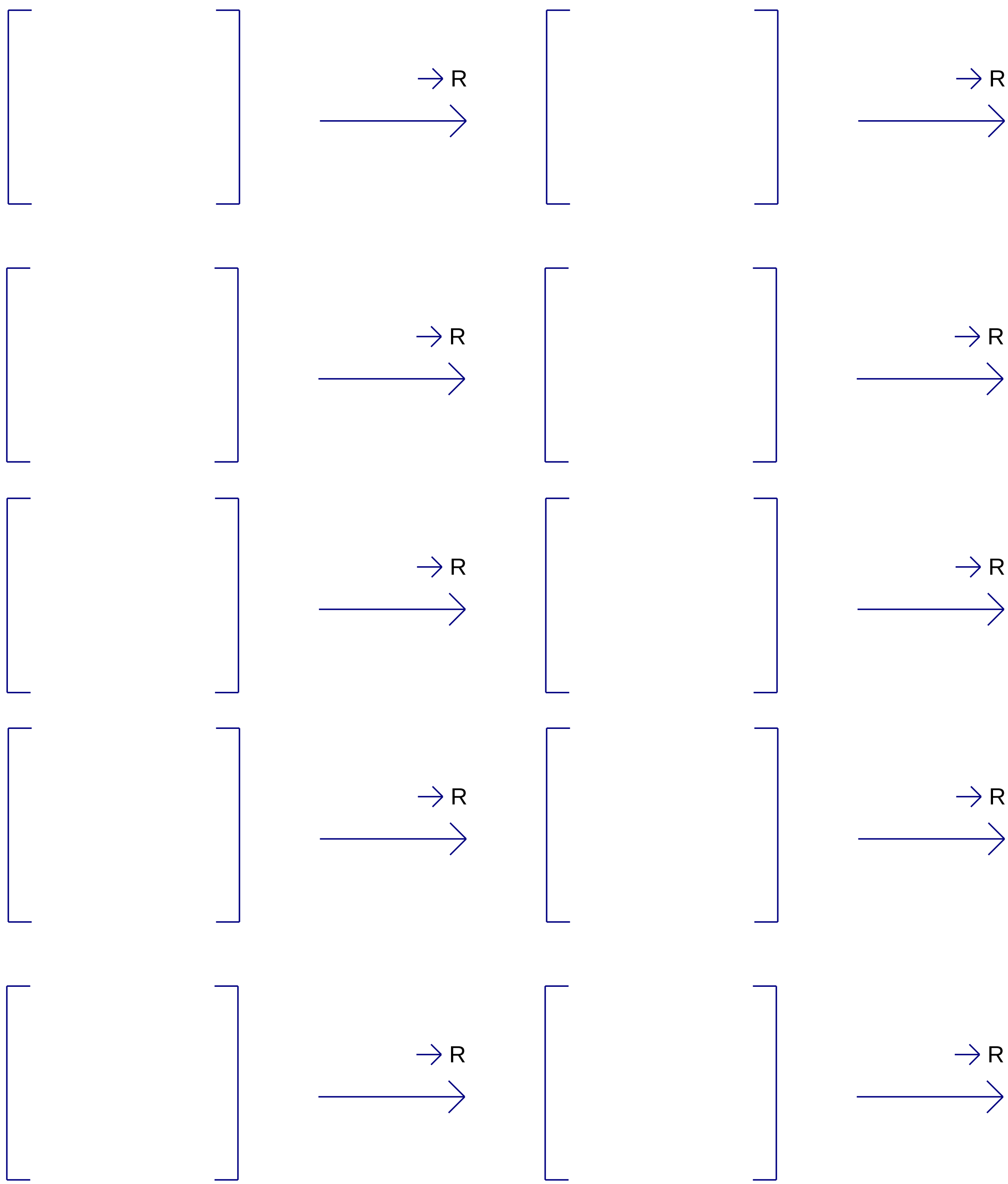
$$\begin{bmatrix} 1 & 1 & 1 & 38 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 28 \end{bmatrix} \xrightarrow{R_1-R_2 \to R_2} \begin{bmatrix} 1 & 0 & 1 & 31 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 28 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 31 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 28 \end{bmatrix} \xrightarrow{R_1-R_3 \to R_1} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 28 \end{bmatrix}$$

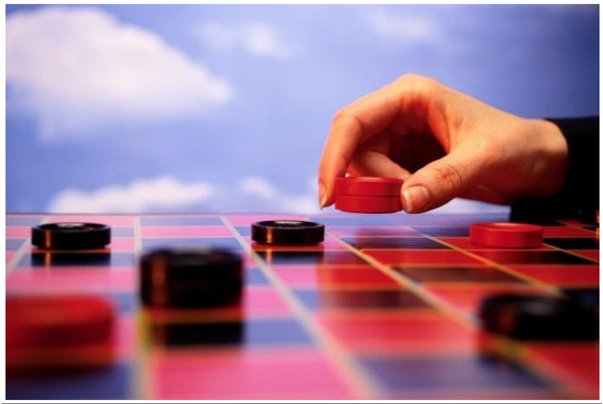
Therefore, the first number is 3, the second number is 7, and the third number is 28.

Template for Student Work:

Name: ____ Date: ________________________________ Homework: ________________________



Check Mate-rix



Solve the following systems of linear equations using the matrix checkerboard.

$$-2x + 4y = 8$$
$$x + 2y = -12$$

Solution:
Place the chips on the board

		-2	4	8
		1	2	-12

-2	4	8
1	2	-12

Step 1 – Switch the Rows to get 1 in the correct place

$R_1 \leftrightarrow R_2$

1	2	-12
-2	4	8

Step 2 – Multiply row one by 2 and add it to row two to create a 0 in the bottom left corner

$2R_1 + R_2 \rightarrow R_2$

1	2	-12
0	8	-16

Step 3 – Multiply row two by $^1/_8$ to create a 1 in the correct place

$\frac{1}{8}R_2 \rightarrow R_2$

1	2	-12
0	1	-2

Step 4 – Multiply row two by -2 to create a 0 in the correct position

$-2R_2 + R_1 \rightarrow R_1$

1	0	-8
0	1	-2

Step 5 – Read the solution

1	0	-8
0	1	-2

$x = -8$

$y = -2$

The two lines intersect at $(-8, -2)$

Little Caesar says: "Pizza! Pizza!"

The last Little Caesar's pizza place in Victor is having a special on 3 kinds of slices of pizza. Dr. Howard is ordering for his van of teacher candidates. He ends up spending $15 on 7 slices of cheese, 1 slice of pepperoni, and 2 slices of supreme. Dr. Cox orders herself 1 slice of cheese, 1 slice of pepperoni, and 2 slices of supreme for later and spends $9. Lastly, Dr. Straight orders 4 slices of supreme, and after becoming sufficiently ill, decides to get 1 slice of cheese and 1 slice of pepperoni to save for the ride home if he gets hungry, which costs him a total of $15. If Amy and Keely have $6 and can't decide what kind to get, would they be able to share one of each slice of pizza?

Solution:
Let x be the cost of the slices of cheese pizza
Let y be the cost of the slices of pepperoni pizza
Let z be the cost of the slices of supreme pizza
Dr. H→ $7x + y + 2z = 15$
Dr. C→ $x + y + 2z = 9$
Dr. S→ $x + y + 4z = 15$

$$\begin{bmatrix}7&1&2&15\\1&1&2&9\\1&1&4&15\end{bmatrix}\xrightarrow{-R_2+R_3\to R_3}\begin{bmatrix}7&1&2&15\\1&1&2&9\\0&0&2&6\end{bmatrix}\xrightarrow{\frac{1}{2}R_3\to R_3}\begin{bmatrix}7&1&2&15\\1&1&2&9\\0&0&1&3\end{bmatrix}\xrightarrow{-2R_3+R_2\to R_2}$$
$$\begin{bmatrix}7&1&2&15\\1&1&0&3\\0&0&1&3\end{bmatrix}\xrightarrow{-2R_3+R_1\to R_1}\begin{bmatrix}7&1&0&9\\1&1&0&3\\0&0&1&3\end{bmatrix}\xrightarrow{R_1\leftrightarrow R_2}\begin{bmatrix}1&1&0&3\\7&1&0&9\\0&0&1&3\end{bmatrix}\xrightarrow{-7R_1+R_2\to R_2}$$
$$\begin{bmatrix}1&1&0&3\\0&-6&0&-12\\0&0&1&3\end{bmatrix}\xrightarrow{-\frac{1}{6}R_2\to R_2}\begin{bmatrix}1&1&0&3\\0&1&0&2\\0&0&1&3\end{bmatrix}\xrightarrow{-R_2+R_1\to R_1}\begin{bmatrix}1&0&0&1\\0&1&0&2\\0&0&1&3\end{bmatrix}$$

$$x = 1$$
$$y = 2$$
$$z = 3$$

Yes, Amy and Keely can share one slice of each type of pizza since the total for one slice of each kind of pizza is $6.