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Tricky Trickery!

Introduction: In this lesson plan is included specific math tricks that can be used to introduce your students into beginning stages of algebra, and to gain the interest of the students at the beginning of class. This lesson plan is intended for students in grades 8-10. By the end of this lesson the students should be able to understand and be ready to learn more on basic algebra topics.

MST Standards:

- 8.A.6 Multiply and divide monomials
- 8.A.7 Add and subtract polynomials (integer coefficients)
- Connections strand- Understand and make connections among multiple representations of the same mathematical idea
- A.R.3 Use representation as a tool for exploring and understanding mathematical ideas.
- Communications- Analyze mathematical solutions shared by others

NCTM Standards:

- use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts
- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules.

Instructional Objectives:

- ✓ Students should be able to understand how these tricks work
- ✓ Students will be able to answer simple algebraic problems
- ✓ Students will be able to perform the tricks on others

Instructional Protocol:

- ➢ This lesson could be used for a Friday fun activity or a before break activity.
- ➢ You could either choose a couple of tricks for your lesson or all of them, depending on what you want to accomplish in the class.
- ➢ The algebra trick will be used to show students that they can make their own magic trick to amaze their friends and family.
- ➢ The Math Mystic tricks could be used to introduce divisibility theorems and the distributive property.
- ➢ The other amazing number tricks can be used to blow the minds of the students during class and at home and enhance mental math.

Math Mystic

As a young lad, I went to visit my great-grandfather every weekend. There were always milk and chocolate chip cookies on the table for me when I walked in. While we were eating the delicious treats he would tell me he could read minds. Sure enough after doing a little work on a four digit number he could tell me the digit I left out after telling him the other digits. After that trick was over, to prove his mind reading capabilities, he would have me do a little multiplication and once again, only leaving out one digit, he could invariably tell me what it was.

Steps:

1) Pick a four digit number.
2) Now, allow the volunteer to rearrange the four digit number in any way he or she desires.
3) Now using the two numbers subtract to obtain a positive number.
4) Tell me any three of the digits, and I will tell you the remaining digit.

The "math mystic" will never look at the numbers used by the volunteer

PROOF:

If we reorganize our numbers in the subtraction problem we will get $abcd - cabd = 1000(a-c)+100(b-a)+10(c-d)+1(d-b)$ to see the 1000's place, 100's place, 10's place, and the 1's place.

By using the distributive property we will get

$=1000a-1000c+100b-100a+10c-10d+d-b$

By combining like terms we obtain this

$=900a+99b-990c-9d$

If we factor out a 9 the result will give us this

$=9(100a+11b-110c-d)$

This shows us that the divisibility by 9 rule, can be used on the result of the subtraction problem.

Most people may not think of subtraction in this way, but it is the inner workings of base 10 subtractions that will yield this answer. Because the result is divisible by nine we can use the number nine divisibility rule to obtain the remaining digit. The way to do this is to add the three digits that were given to you in the trick and subtract from the next multiple of nine.

EXAMPLE:

The volunteer chose the four digit number 8965

Rearranged the number to 5986

The result of the subtraction is 2979

The three digits given are 9, 7, and 9. The sum of these numbers is 25. When subtracted from the next multiple of nine, 27, we obtain 2.

However in the special case that the number left out is a 9 or 0 is where the magician goes to work. Say the three numbers given are 9, 7, and 2. The sum is 18 so our choices of available numbers are either 0, because 18 is a multiple of 9, or 9 because the next multiple of 9 is 27. At this point the magician should guess at the number, and if it is wrong play it off as the volunteer is not thinking of the number hard enough, or that someone else is interfering with your thought process.

THE CONSECUTIVE CRAZINESS!

STEPS:

1) Pick three consecutive numbers and multiply them together.
2) Pick three more consecutive numbers and multiply them together.
3) Multiply the two products together.
4) Pick one digit of the resulting product and tell me the other digits, and I will tell you the remaining digit.

PROOF:

Any three consecutive numbers multiplied together will result in the product being divisible by three. This is true because one of the three consecutive numbers chosen will be a multiple of three. Therefore when two sets of three consecutive numbers are multiplied together the resulting number is divisible by 9. Now since, the result of the product of the two sets is divisible by 9, the divisibility 9 rule will again allow us to obtain the last remaining digit, when the rest are given.

EXAMPLE:

The two sets of three consecutive numbers are 4, 5, and 6 and 8, 9, and 10. So the product of the first set is 120 and the second is 720. So when multiplied together we obtain 86400. Now, no matter what numbers they give us we can obtain the last digit using the same ideas as the last trick. The special case still applies here.

Age Arithmetic

The magic number 115

Without even looking at the paper the volunteer is working on, and not knowing anything about said volunteer, the magician can tell the volunteer what month they were born in AND how old they are.

Ask your volunteer to write down the number of month in which s/he was born. (January is 1; February is 2 and so on).

Turn your back so you cannot see what your volunteer is jotting down in response to your instructions. Then have that person write down:

Month of birth (Example February): 2
Double it: 4
Add 5: 9
Multiply by 50: 450
Add volunteer's age (Example 40): 490
Subtract 365: 125

At this point, ask your volunteer to call out the total, which in this case is 125.
In your head, quickly and secretly add 115.
In this example, you will end up with 240.
Then proclaim, "Why, you were born in February and you're 40 years old."

Here's how you did it. The first number -- and the second if the total is a four-digit number -- will indicate your volunteer's birth month, once you've added good old 115. The last two digits will indicate his/her age.

This trick will work no matter what the age or month of birth because you have good old 115 on your side.

Why don't you use some algebra to prove it?
Let m = month and a = age.
Then following the instructions above:
Month of birth: m
Double it: 2m
Add 5: 2m + 5
Multiply by 50: 100m + 250
Add friend's age: 100m + 250 + a
Subtract 365: 100m - 115 + a
Add 115 to get: 100m + a. (The 100 just moves the month over 2 places)

What year is it?

Don't we all have some year in our lives that we think was really important? Maybe you have more than one. Let's see a little something that may pique some interest.

Write down the year in which you were born.
Write down an important year during your lifetime.
Next, write down the number of years that have elapsed since that important year.
Put down your age this year on your birthday.
Add up the numbers on your list.
Divide by 2.

If your mental computer is working, you should have the current calendar year.

Let's take a look at an example.

Say someone is born in 1960 and they believe an important year for them is 1980 (the year he/she bought his/her first car). The number of years that have elapsed since that important year (this year is 2007) is 27. The age of this person is or will be this year is 47. Add all these up and you will get 4014. Divide by two and you get 2007! Peculiar…

Let's look at the algebraic reason why this works. Let's assign these values some letters to make it easier to see.

Let x be the current year.
Let y be the year that the volunteer is born.
Let i be the important year for that person.

What exactly did we have them do?

The age of a particular person is just really the current year minus the year they were born, or x-y.

The number of years that have passed since the important date can be written in a similar way, x-i.

So, let's do the calculation.

We want to add the year they were born, an important year for them, the age of the person this year on his/her birthday, and the number of years that have passed since the important year. Let's add these together using our variables.

$y+i+(x-y)+(x-i) = 2x$

Dividing this by two yields x, so no matter what happens we will always get the current year!

"Always Get 13"

1. Pick a number
2. Add 10
3. Multiply that number by 100
4. Subtract 500 from that number
5. Divide by 50
6. Add 16 to your new number
7. Cut it in half
8. Absolute value of the difference between your original and current numbers.
9. Answer should be 13

Try it out with **9**

1. 9
2. 19
3. 1900
4. 1400
5. 700
6. 28
7. 44
8. 22
9. **22-9= 13**

Here is the **general** case

x
x+10
100x+1000
100x+500
50x+250
2x+10
2x+26
x+13
x-x+13

Try it out with a bigger number **89**

1. 91
2. 101
3. 10100
4. 9600
5. 4800
6. 192
7. 208
8. 104
9. **104-89=13**

This trick will always work for any number. And you can morph the math to end with any number.