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No Pi Allowed

Introduction:

Students often confuse or forget the area and circumference formulas for circles. In addition, students rarely are given the opportunity to derive such critical formulas. Below are four different activities you can incorporate in your classroom to derive the formula for the area of a circle:

1. Pie Tastes Better Than π.
2. Marble Madness!
3. Hitting the Target!
4. No π, No Weigh!

NYS MST Standards:

- 6.R.7 Use mathematics to show and understand physical phenomena (e.g., determine the perimeter of a bulletin board).
- 6.G.6 Understand the relationship between the diameter and radius of a circle.
- 6.G.7 Determine the area and circumference of a circle, using the appropriate formula.
- 6.M.7 Estimate volume, area, and circumference (see figures identified in geometry strand).

NCTM Standards:

- Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.
- Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.
- Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision.

Instructional Objectives:

Objectives of this lesson include:

- Introduce fun ways of calculating the area of a circle.
- Provide a solid base understanding of the area of a circle.
- Introduce estimation of the area of a circle.

Instructional Protocol/Itinerary:

- The audience will be asked questions before presenting each method to spark their interest and get their minds thinking about circles.
- The lessons will be hands on and will involve the students.
- All lessons will be designed as a visual aide to understanding the concepts behind finding the area of a circle.
- Lessons can be altered to use different models appropriate for the setting.

<u>**Pie Tastes Better Than π**</u>

Why is it that the area of a square or a rectangle is base x height but the area of a circle is πr^2? Wouldn't it make sense to have just one formula for area? Well actually, we do. It is base x height. Now this might get a little bit confusing, so since it's getting to be that beautiful fall season again, here is a way to help illustrate why the area of a circle is equal to base x height.

Why can't we use base x height to calculate the area of a circle?

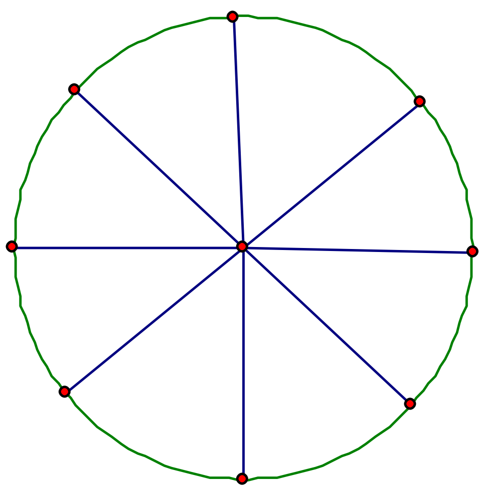
We don't know what the base and the height of the circle are.

In that case, what we are really trying to find is a way to show that:

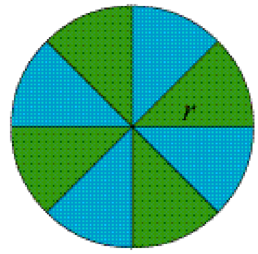
$$\pi r^2 = base \times height$$

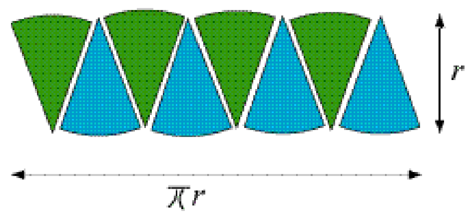
Here is a step by step process to help us see this more clearly, and give us a tasty treat:

Step 1: Start with a delicious pumpkin pie with a diameter of 9 inches. Cut the pie into 8 equal slices.



Step 2: Remove slices from the pie tin and arrange them as a parallelogram.





Step 3: Now we have a parallelogram. The area of a parallelogram is base x height. Measure the base and the height of the parallelogram in inches. Notice that the height of the parallelogram is also the radius of the pie, and the base is one half the circumference.

The base of the parallelogram was 14 inches and the height was 4 and a half inches.

Step 4: Find the area of the parallelogram:

$$A = base \times height$$
$$A = 14in \times 4.5in$$
$$A = 63in^2$$

How close are we?

Now compute the area of the circular pie using πr^2.

$$A = \pi r^2$$
$$A = \pi \times (4.5in)^2$$
$$A = 20.25\pi in^2$$
$$A \approx 63.617in^2$$

Why does this work?

$$A = base \times height$$

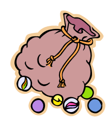
$$A = \frac{1}{2} Circumference \times radius$$

$$A = \frac{1}{2}(2\pi \times radius) \times radius$$

$$A = \pi \times (radius)^2$$

*MODIFICATIONS OF THIS ACTIVITY:

- You can have students cut paper plates and form the parallelogram. Then they can find the area of the parallelogram and derive the formula for the area of a circle.



Marble Madness!

Let's face it: some students do not like circles and their formulas. Marble Madness is all about manipulating the area of a circle into a "more pleasant" area that we can work with, namely, a rectangle. Let's see how this works:

How can we approximate the area of a circle without its dimensions or using the formula? One possible method would be using marbles. Just follow these four easy steps.

Step 1: Take your circular cake pan, and fill the bottom with one flat layer of marbles.

Step 2: Transfer your Marbles to the rectangular cake pan. Position the marbles such that you have only one layer, and the marbles are packed as tightly as possible.

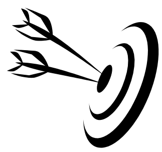
Step 3: Note how much of the cake pan is filled by the marbles.

Step 4: Now actually measure the dimensions that the marbles fill in the rectangular pan. Apply the area formula for a rectangle, and you have your estimate for the area of a circle! (To compare this value to the actual area of the circle, measure the radius of the circular pan, and apply the formula for area of a circle.)

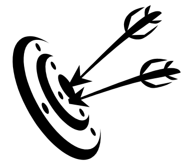
*MODIFICATIONS AND OTHER APPLICATIONS OF THIS ACTIVITY:

- Instead of marbles, feel free to use other small objects to take up the area; candy works great!
- You can also use this activity to find an approximation of π. Create the rectangular pan or box such that the radius of the circle is the height (h=r), and four times the

radius of the circle is the base (b=4r). Once you transfer the marbles and measure to where they fill, you will note that the marbles fill to a little over $\frac{3}{4}$ of the base. Thus applying the area formula for rectangles, we obtain A= $\frac{3}{4}$(4r)(r) = $3r^2$. Comparing this to the formula for the area of a circle, we see that $\pi \approx 3$.

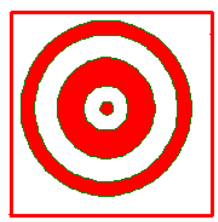


Hitting the Target!



How does playing darts relate to determining the area of a circle? Well, let's find out how on target you and your classmates are.

THE SETUP:
Create a square circumscribed about the dart board (this can be done by taping off a square closely around the dart board). Now you are ready to fire away.



THE RULES:

- Each student will be allowed three chances to hit the target from a fixed distance away from the dart board.
- Each student is to tally his/her hits and misses as well as other classmates' hits and misses in the given table.

How to Get a Hit:

- The dart is counted as a hit only when it lands and sticks inside the circle.
- If the dart lands inside the square but not in the circle, it is counted as a miss.
- If the dart lands outside of both the circle and square, it is not counted.

HITS	**MISSES**
TOTAL HITS:	TOTAL MISSES:

TIME FOR THE MATH:

Now time for the fun part! Based on the total hits, we can determine an estimate for the area of our circular dart board, no π allowed! Here's how:

- Setup the following proportion:

$$\frac{TOTALHITS}{HITS + MISSES} = \frac{ESTIMATEOFCIRCLEAREA}{AREAOFSQUARE}$$

- Fill in the known information (total hits, hits plus misses, and the area of the square), then solve the proportion for our unknown value, the estimate area of the circle (dart board). (The total hits and hits plus misses can be calculated by simple addition. We can physically measure the length and width of our square and compute the area).
- Once we have our estimate, we can then compare that value to the exact area of the circle by implementing the area formula for a circle, $A = \pi r^2$. (We can physically measure the radius of the circle).

WERE WE ON TARGET?

Discuss the following questions:

1. Was our estimated area of the dart board a good or bad estimate? Why?

   SOLUTION: This depends on your data. If you had a reasonable amount of hits compared to the total amount of darts thrown, your area estimate for the circle should be close. If for some reason every dart was a hit or you had a substantial amount of misses, then your area estimate will have a higher percent error.

2. Is there anyway to provide a better estimate using this method? Explain.

   SOLUTION: Yes, because the more trials you have, the closer your estimate should be. The ratio should become more exact.

3. Why does this proportion seem to work?

   SOLUTION: This has to do with the concept of area and the probability of hitting the circle. The way we circumscribed the square around the dart board made for the circle to cover most of the same area of the square. Thus the probability of hitting the circle is equivalent to the area of the circle over the area of the square. Notice that every hit in the circle also is a hit in the square, but every hit in the square is not necessarily a hit in the circle. Therefore, when tossing darts, more of them seem to land in the circle because this covered most of the square, which relates to our concept of area.

*MODIFICATIONS OF THIS ACTIVITY:
Instead of using darts, this activity can be done by:

- Using a circle inscribed in a square grid divided into 100 parts, each part labeled in coordinate fashion from 00 in the lower left corner to 99 in the upper right corner. You then can use a random number generator to plot points according to the coordinates (a hit in this case would be if the point was plotted inside the circle). Apply the same proportion and follow up questions.
- Have a hula-hoop lie flat on the ground; use masking tape to tape off a square around the outside of the hoop. Use the same chart, and have students toss bean bags into the hoop from a fixed distance away (a hit will be counted if the bean bag lands and stays inside the hula-hoop). Apply the same proportion and follow up questions.

<u>No π, No Weigh!</u>

Can we really estimate the area of a circle by its weight? I think so… and here is what you will need:

Materials:

- Circular cutout of a floor tile with the diameter of 21.5 cm (Trace a pie tin).
- Three 10 cm x 10 cm squares of linoleum.
- Five 10 cm x 1 cm rectangles of linoleum.
- Ten 1 cm x 1 cm squares of linoleum.
- Scale (preferable a mass balance scale, but any scale will work).

I still don't get it? How is this going to work?

Procedure:

- Let's take our circular cutout and place it on one of the balances. Record the weight.
- Now let's try to match the weight of the circular cutout with our four sided weights.
- Try to come as close as possible to achieve the same weight as the circular cutout by combining various square or rectangle tiles.

So, the weight of the circular cutout and the combined weight of the various squares and rectangles came pretty close in number. What's that have to do with area?
The Magic behind the Mystery:

- Take a look at your circular cutout and the weights you ended up using to balance their mass.
- Try to arrange the weights you used to resemble a circle. Any thoughts yet?
  - *The scale should balance when the weights covered exactly the same area as the circular region!*
- You could also compute out the areas of both and then compare:
  - For the total area of the weights, sum together each individual area of the rectangles and squares using $A=b\times h$.
  - For the area of the circular region, measure the radius (if not known), then apply the formula for area of a circle, $A=\pi r^2$.
  - Compare the two areas; I bet that you'll find they are relatively close. NO WEIGH!