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"To Tiles and Beyond"

Introduction: This lesson explores a visual, hands-on algebra manipulative, specifically algebra tiles, applicable to both Algebra and Algebra 2. It is designed to incorporate a grid structure into multiplication of binomials, and then extends to the use of this grid multiplication to any polynomial with the use of trinomial multiplication. Then, an activity using algebra tiles to represent the process of completing the square is introduced in a new light. Finally, the lesson explores the correlation between geometry (algebra tiles), completing the square to solve quadratics, and a derivation of the Quadratic Formula.

NYS MST Standards Addressed:

- 8.A.8 Multiply a binomial by a monomial or a binomial (integer coefficients).
- A2.A.24 Know and apply the technique of completing the square.
- A2.RP.1 Support mathematical ideas using a variety of strategies.

NCTM Standards Addressed:

- Understand the meaning of equivalent forms of expressions, equations, inequalities, and relations.
- Use geometric models to represent and explain numerical and algebraic relationships.

Instructional Objectives:

- Introduce students to different ways of multiplying polynomials.
- Connect various methods of solving quadratic equations.
- Allow students to communicate mathematically, both verbally with group members and in writing.

Instructional Protocol/Itinerary:

- Make sure to illustrate that in the Football area problem that the algebra tiles must match up with the row and column. This will be a good segue into the Grid Multiplication Activity. Also, make sure to have blank "grid" worksheets so that students can place their tiles on them accordingly.
- In the Multiplying Trinomials section, show students that they can add up the diagonals in the grid which represent the like terms. Note that this is similar to lattice multiplication.
- Use the 1st page of the Tiling the Square Activity as a introduction to completing the square. Then let the groups work on the 2nd page as you observe and help as needed.
- In the Extension section, use the sheets as guided notes, while modeling computations on the board.

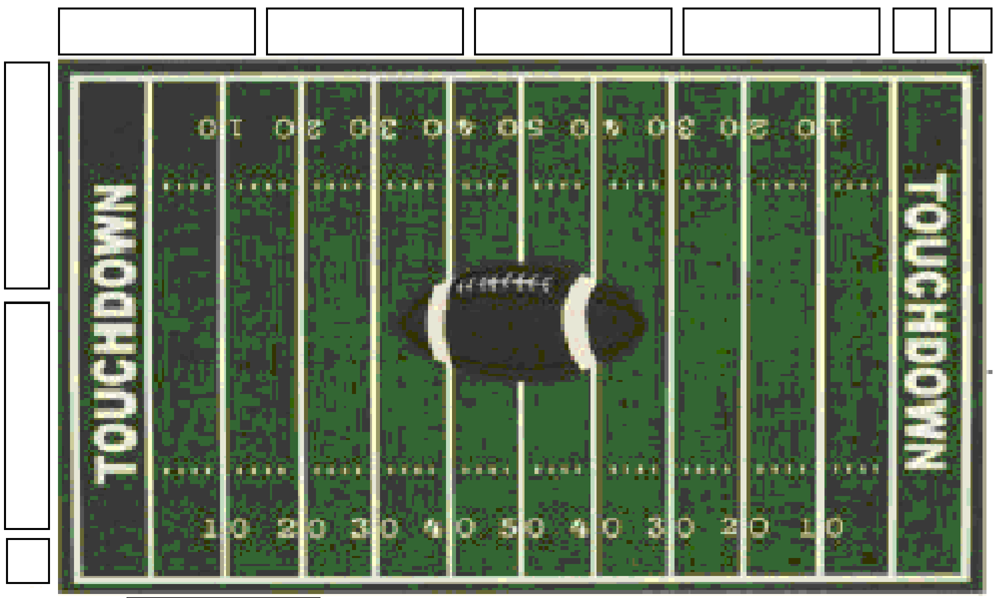
Required Materials: Algebra Tiles for each group and Overhead tiles, Grid Worksheets, and Guided Notes Worksheets.

<u>Falconer Football</u>

It's October and high school football is back. The coach challenged our math class to figure out what the area of the field is. The coach estimated the dimensions of a football field in yards^2. There's only one problem, he decided to give the dimensions to the coach as $4x + 2$ yards by $2x + 1$ yards. Thus, we must figure out the area in terms of x.

To do this, fill in the football field below with Algebra Tiles. Make sure that the edge lengths match up to the tiles in each row or column.

$4x + 2$



$2x+1$

Key: x^2 = x = 1 yard^2 =

Now, add up the amount of each kind of tile you have. This should represent the area of the football field in terms of x.

Area =

Finally, the coach told us that $x = 29.5$ yards. So what is the actual area in yards^2?

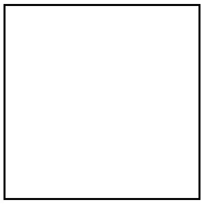
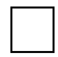
Grid Multiplication with Tiles

Let's see how we can multiply binomials to obtain trinomials using a new technique.

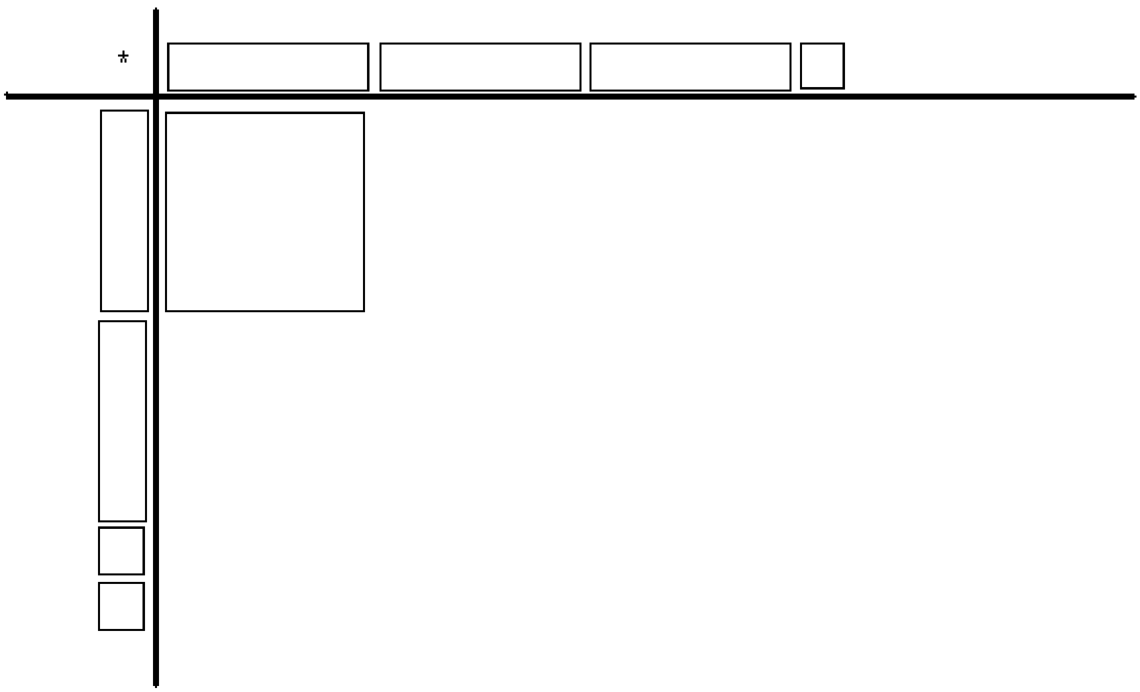
Example 1:

Consider 2 binomials (2x + 2) and (3x + 1).

Setup: Use algebra tiles and grid worksheet to set up a multiplication grid as follows.

 Represents an x^2.  Represents an x.  Represents a 1

1.



Use each tile as a row or a column to multiply. For example in the first row, first column, you would get x * x = x^2. Then place an x^2 tile in the appropriate spot.

Fill in the rest of the chart.

Then, record your findings below.

2.________________Number of x^2 tiles. Now write the product below:

3.________________Number of x tiles.

4.________________Number of 1 tiles. 5.________________________________

Example 2:

Note: Black Tiles represent negatives.

When multiplying negatives, remember to make the signs agree.

For Example: -x * 1 = -x so make sure to use a negative x tile.

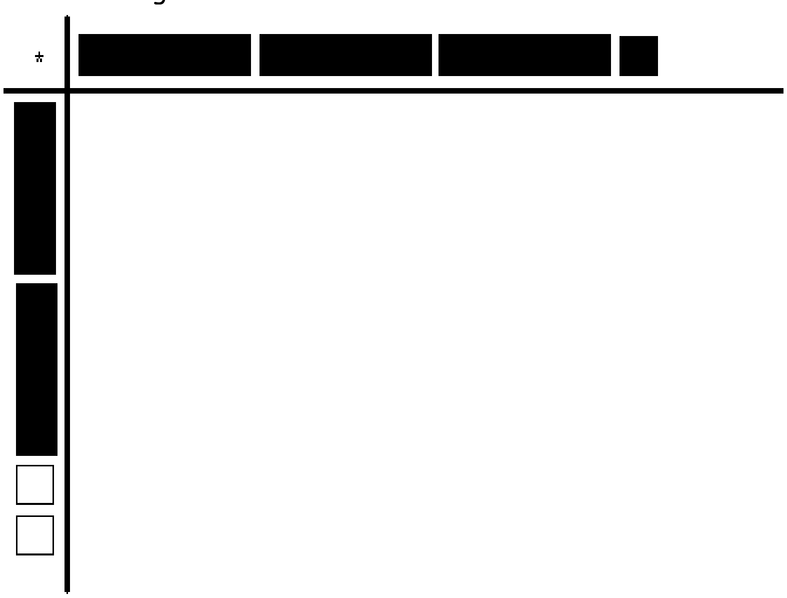
Work in groups to fill in the chart and answer questions.

6. What are the factors? _______________ and ______________.
7. Fill in the grid on your worksheet.

8. _______________Number of x^2 tiles. Now write the product below:
9. _______________Number of x tiles.
10. _______________Number of 1 tiles. 11. ______________________________

Example 3:

12. Factors: ______________ and _____________.
13. Fill in the grid.



14. Final product: ______________________

Multiply Trinomials!!!

Example 4:

Fill in the charts the same way we did when we used algebra tiles.

Remember, when multiplying exponents to add the exponents.

15.

*	$2x^2$	$4x$	3
$-3x^2$			
$5x$			
-5			

Combine like terms by adding the coefficients.

16. Coefficient of x^4 terms: ______________.19. Coefficient of x terms:______________.
17. Coefficient of x^3 terms:______________.20. Sum of constant terms:_____________.
18. Coefficient of x^2 terms:______________.

Use 16.-20. to write your final product.

21. _____________________________

Example 5:

22. Fill in the grid.

*	$-x^2$	$\frac{1}{2}x$	3
$-4x^2$			
$8x$			
-2			

Combine like terms by adding the coefficients.

23. Coefficient of x^4 terms: _____________.26. Coefficient of x terms:______________.
24. Coefficient of x^3 terms: _____________.27. Sum of constant terms:_____________.
25. Coefficient of x^2 terms:______________.

Use 23.-27 to write your final product.

28. _____________________________

<u>Tiling the Square</u>: Group Activity

Here's a new look at a different approach to solving quadratics. In the 9th Century C. E., a Babylonian mathematician named Al-Khwarizmi was the principle creator of the process that is now known as completing the square. We'll use algebra tiles to understand how geometry can be used to solve algebraic equations.

Consider the quadratic equation $x^2 + 6x - 4 = 12$.
1. Write the equation in $ax^2 + bx = -c$ form.

2. Arrange the 6"x" tiles around the 1 "x^2" tile to form part of a square. What did you have do to the 6 "x" tiles in order to make the partial square?
Draw your results.

3. Now that you have a partial square, how many "1" tiles can you fit to complete the square?

____

4. What are the dimensions of our completed square?

___________________ and ________________ or _________________

5. So, now rewrite the left hand side of your equation by substituting your completed square for the perfect square trinomial you have created in step 3.

6. Solve for x.

Work in groups to solve the following equations by completing the square.

a. Write the equations so that the coefficient of the x^2 term is 1.
b. Make sure that your equation is in $ax^2 + bx = -c$ form.
c. Use the correct amount of algebra tiles to arrange tiles into a partial square.
d. Complete the square with "1" tiles and rewrite the equation.
e. Solve for x.

7. $4x^2 + 32x = 132$

a.

b.

c.

d.

e.

8. $2x^2 + 20x - 6 = 16$

a.

b.

c.

d.

e.

Extension

Consider the equations on your group worksheet as $ax^2 + bx = -c$

We wanted to make sure that a = 1, so we divided through by a to get $x^2 + \frac{b}{a}x = -\frac{c}{a}$.

Now, when we look at b in Problem 9, $\frac{b}{a} = 8$.

What relationship do we see that corresponds to the sides of our completed square? In other words, if (x + d) was the side of our square, what is the relationship between $\frac{b}{a}$ and d? (Hint: d = ?).

Notice that $\left(\frac{b}{2a}\right)^2$ is the number of "1" tiles we added to complete the square each time.

****Also, do not forget that we added this to both sides.

So, could we use this to complete the square without algebra tiles? Yes.

If we add $\left(\frac{b}{2a}\right)^2$ to both sides, what does the equation become?

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

At this point we rewrote the equation to get:

$$(x - d)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

Or

$$\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

If we simplify the right hand side of the equation, we now have:

$$\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b^2 - 4ac}{4a^2}\right)$$

Then we took the square root of both sides to get:

$$x + \frac{b}{2a} = \pm\left(\frac{\sqrt{b^2 - 4ac}}{2a}\right)$$

Now if we solve for x, the equation becomes

$$x = \left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

This formula is what is known as the Quadratic Formula.
***Remember, to use the Quadratic Formula, the quadratic equation you are solving must be in standard form ($ax^2 + bx + c = 0$).

Now we have three ways of solving quadratic equations in the form: $ax^2 + bx - c = 0$.
I.

II.

III.

Answer Key.

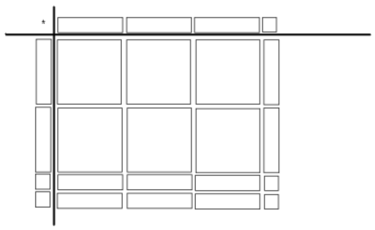
Falconer Football

Area = $8x^2 + 8x + 2$

Let x = 29.5 yards, then Area = 7200 $yards^2$

Grid Multiplication with Tiles

1.



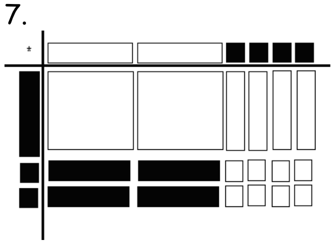
2._________6________Number of x^2 tiles.   Now write the product below:

3._________8________Number of x tiles.

4._________2________Number of 1 tiles.   5. $6x^2 + 8x + 2$__________________

6. (2x – 4) and (-x – 2)

7.



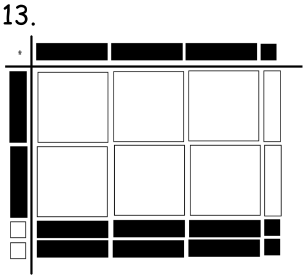
8. _________-2________Number of x^2 tiles.   Now write the product below:

9. _________0________Number of x tiles.

10._________8________Number of 1 tiles.   11. $-2x^2 + 8$__________________

12. Factors: _____-3x - 1_____ and ____-2x + 2________.

13.



14. Final product: ____$6x^2 - 4x - 2$____________________

Multiply Trinomials!!!

15.

*	$2x^2$	$4x$	3
$-3x^2$	$-6x^4$	$-12x^3$	$-9x^2$
$5x$	$10x^3$	$20x^2$	$15x$
-5	$-10x^2$	$-20x$	-15

16. $-6x^4$ 17. $10x^3 + -12x^3 = -2x^3$ 18. $-10x^2 + 20x^2 + -9x^2 = x^2$ 19. $-20x + 15x = -5x$ 20. -15

21. $-6x^4 - 2x^3 + x^2 - 5x - 15$

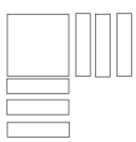
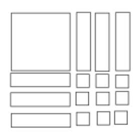
22.

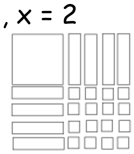
*	$-x^2$	$\frac{1}{2}x$	3
$-4x^2$	$4x^4$	$-2x^3$	$-12x^2$
$8x$	$-8x^3$	$4x^2$	$24x$
-2	$2x^2$	$-x$	-6

23. $4x^4$. 24. $-8x^3 + -2x^3 = -10x^3$ 25. $2x^2 + 4x^2 + -12x^2 = -6x^2$ 26. $-x + 24x = 23x$ 27. -6

28. $4x^4 - 10x^3 - 6x^2 + 23x - 6$

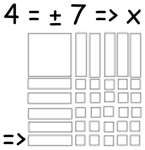
Tiling the Square Activity

1. $x^2 + 6x = 16$
2.  Split them in half. 
3. 9 "1" Tiles
4. __x + 3___ and ______x + 3_____ or ______$(x + 3)^2$____________
5. $x^2 + 6x + 9 = 16 + 9 \Rightarrow x^2 + 6x + 9 = 25$
6. $(x + 3)^2 = 25 \Rightarrow x + 3 = \pm 5 \Rightarrow x = -8, x = 2$



7. $4x^2 + 32x = 132 \Rightarrow x^2 + 8x = 33 \Rightarrow$

$\Rightarrow x^2 + 8x + 16 = 33 + 16 \Rightarrow (x + 4)^2 = 49 \Rightarrow x + 4 = \pm 7 \Rightarrow x = -11, x = 3$



8. $2x^2 + 20x - 6 = 16 \Rightarrow x^2 + 10x - 3 = 8 \Rightarrow x^2 + 10x = 11 \Rightarrow$

$\Rightarrow x^2 + 10x + 25 = 11 + 25 \Rightarrow (x + 5)^2 = 36 \Rightarrow x + 5 = \pm 6 \Rightarrow x = -11, x = 1.$

Extension

$d = \frac{b}{2a}$.

I. Factoring, if possible.
II. Completing the square by rewriting the equation and solving for x.
III. Using the Quadratic Formula.