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## Tanegrid

### Introduction:

This lesson involves tangrams and ways they can be applicable to topics in mathematics. The audience will learn the steps to constructing a tangram and will utilize different ways to use them to find area.

Methods for calculating area will include:

1. Counting unit boxes
2. Arranging geometric shapes

In addition, more applications are included in the lesson plan.

### NYS MST Standards:

*G.G.29.* Identify corresponding parts of congruent triangles

*G.G.38.* Investigate, justify, and apply theorems about parallelograms involving their angles, sides and diagonals.

*G.G.67* Find length of a segment, given its endpoints.

*6.G.2.* Determine the area of triangles and quadrilaterals (squares, rectangles, rhombi, and trapezoids) and develop formulas.

*6.G.10.* Identify and plot points in all four quadrants

*6.G.11* Calculate the area of basic polygons drawn on a coordinate plane

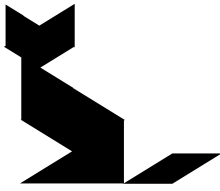
### Instructional Objectives:

Objectives of this lesson include:

- Introducing a fun way to make tangrams and use them to find area.
- Observe the relationship between the individual areas and the entire area.
- Promoting ideas for how to extend this activity to higher grade levels.

### Instructional Protocol:

- The audience will construct tangrams.
- The history of tangrams and relationships between all of its shapes will be discussed.
- The lesson will involve having students discover the formulas for area.
- Students will see links between two areas of mathematics: algebra and geometry.
- This lesson can be revised to make appropriate for grades 9-12.



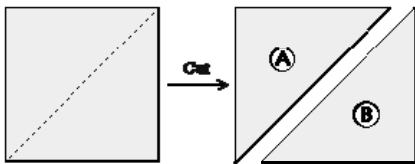
## Tangram History

Tangrams are simple figures that involve cutting seven shapes from a square. When arranged correctly, they must touch and no shapes can overlap. Tangrams are ancient puzzles. They are believed to have been discovered during the Song Dynasty, according to old Chinese records, however the truth still remains a mystery. Legend says that one of the servants of the Chinese emperor was carrying an exquisite ceramic tile around the empire but accidentally tripped, shattering the tile. The Emperor's servant quickly attempted to re-assemble the tile but couldn't. What he did find was that many other shapes resulted that could be formed using the pieces. All the pieces together form a square with 4 equal sides. Upon breaking the tile, the square shattered to form 5 triangles, a parallelogram and a square. The 5 triangles are right isosceles triangles, so they all contain right angles and have two congruent sides. Two sets of the triangles are congruent.

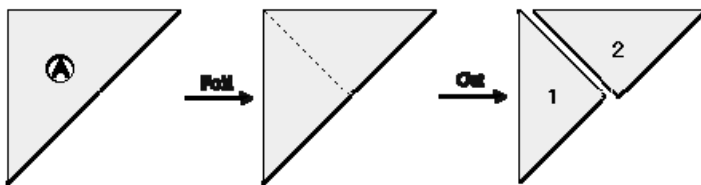
Tangrams can be used to study many applications of geometry, which is what we will accomplish in this lesson. We will use them to arrange a square and calculate areas and calculate the areas for different shapes formed. The area of each individual shape will be calculated along with the area of the entire enclosed figure. Methods of calculating area will be discussed and a comparison will be made at the conclusion of the lesson.

**To start, we need to construct a tangram.**

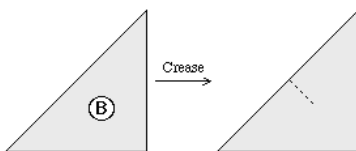
1. Begin with a square piece of paper. Fold it in half along the diagonal and cut along the resulting crease. By cutting these, you have formed 2 congruent right isosceles triangles.



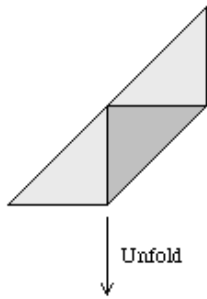
2. Fold one of the resultant triangles in half and cut. Set these two pieces off to the side.



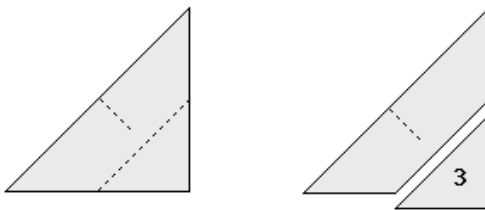
3. Take the other triangle and find the midpoint of the hypotenuse by folding and lightly creasing.



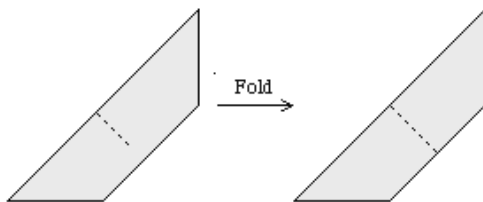
4. Fold the triangle so the vertex of the right angle touches the midpoint of the hypotenuse.



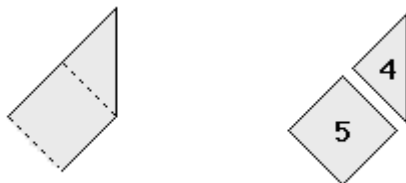
Cut along the crease and set this triangle aside.



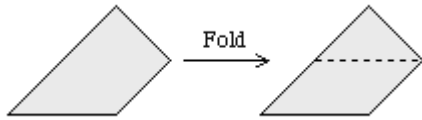
5. Fold the remaining isosceles trapezoid in half along its line of symmetry and cut it into two smaller right trapezoids.



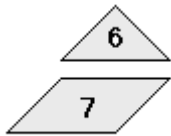
6. Fold one trapezoid so that the vertices at the ends of the longest edge meet. Cut along the crease to make a small triangle and a square.



7. Fold the other trapezoid so that the obtuse angle meets the right angle at the opposite vertex.



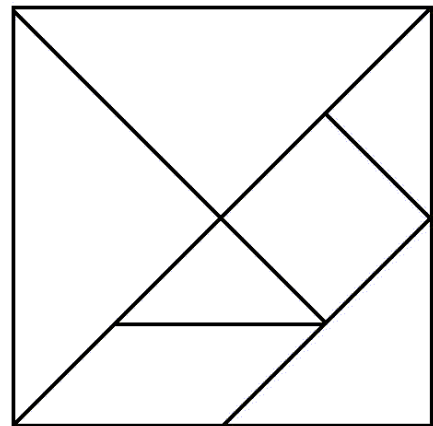
8. Cut along the crease to make another small triangle and a parallelogram.



{Instructions from: <http://mathforum.org/trscavo/tangrams/construct.html>,  
<http://www.ac.wvu.edu/~mnaylor/tangrams/tangramactivities.html> }

**You have now constructed the seven pieces of a tangram!**

The picture at the right shows what your folds would look like if you hadn't cut the shapes out. We are going to use these pieces to help understand the concept of area. We will find the area of the whole tangram and the area of some of the individual shapes. The first method covered will involve counting unit squares.



**Count the Squares!**

When thinking about area, we think about the number of squares required to cover a shape or figure completely, like tiles on a floor. The most basic way to calculate the area of a geometric figure involves counting the squares within the outlined geometric figure. Here's how!

1. Take the tangram and arrange it back to its original formation, making the original square.
2. Place the square with all the pieces correctly arranged onto your axis.
3. Trace each piece onto the grid.
4. Now simply count the number of unit squares inside each individual figure.  
 The number of squares you counted represents the area of that shape.  
 Remember that area is measured in units<sup>2</sup>, so don't forget your units!

**Questions:**

Using the directions above, what is the area of the smallest square?

What is the area of the large triangle?

Can you calculate the area of all the tangrams? How?

What is the area of the large square that contains all the shapes?

- From this, we can see that a very visual way to determine area is by counting the number of enclosed squares. However, this is not the only way in which area can be calculated. For reference and further use, here are the areas of each of the shapes.  
 Large triangles= $50 \text{ units}^2$   
 Square= $25 \text{ units}^2$   
 Smallest triangles= $12.5 \text{ units}^2$   
 Medium triangle= $25 \text{ units}^2$   
 Parallelogram= $25 \text{ units}^2$

Answers: 25, 50, yes-by adding all the boxes, 200

## Move 'em Round

In the activity we just completed, we used the concept of counting unit boxes to determine all the areas of the geometric figures. Using our calculations from above, we will manipulate some pieces of the tangram to simplify the notion of area. Using this method, students will be able to find areas of unfamiliar shapes based on shapes they are more familiar with.

1. Begin by arranging the 2 smallest triangles and the parallelogram.
2. From these pieces, we can construct a rectangle.
3. From here, students can easily calculate the area since they are already familiar with the area of rectangles.

Now look at your 7 geometric shapes. Is there any method to easily obtain the area of the two largest triangles?

Yes there is! Notice that the 2 largest triangles can be arranged to form a square. We can easily calculate this area by observing each side length is 10 units, so the area of the square is  $100 \text{ units}^2$ . Since the square is composed of two congruent triangles, we can then calculate each of their areas to be  $50 \text{ units}^2$  by dividing the total area of the square by 2.

With the remaining five pieces, we can construct another square. Since this square has side lengths of 10 units and an area of  $100 \text{ units}^2$ , we know it is congruent to our other square.

By simply calculating two areas, we know the total tangram has an area of  $200 \text{ units}^2$ . This is a nice shortcut that saves us from the tedious task of finding seven different areas and adding them together. This answer,  $200 \text{ units}^2$ , is the same area we calculated in the activity above, so we can conclude that both methods will provide us with the same answers.

## Dare to Compare?

In both methods above, we have calculated area in terms of units. However, now we will find area by making a comparison to the individual pieces and the entire tangram.

We can observe that the five-piece square we arranged above represents  $\frac{1}{2}$  of the total tangram area.

If we break it up and look at the smallest square, what relationship can we see when it is compared to the five-piece square?

We can see that it represents  $\frac{1}{4}$  of the total five-piece square and would represent  $\frac{1}{8}$  of the total area, calculated to be 200 units<sup>2</sup>.

- If we put the two smallest triangles together, we can form another small square. Now look at the largest triangle. We know two of the legs of the triangle measure 10 units. If we were to halve these, we would have a triangle with two legs measuring 5 units. The smaller triangles have an area that is  $\frac{1}{4}$  of the area of the large triangle.

This leads us to the conclusion that when the length of the sides is doubled, the total area is quadrupled. By making comparisons and looking at relationships, we can calculate the area of each individual shape and the area of the tangram.

## All Tied Together

During this lesson, we have been successfully able to find the area of geometric shapes without ever mentioning the word "formula." Based on the calculations, the students can derive the formulas for area themselves.

Refer back to the area we calculated for the square; 100 units<sup>2</sup>.

- This square was made up of two congruent triangles, each with an area of 50 units<sup>2</sup>.
- We knew each had leg lengths of 10 units.
- We can write this as a simple algebra problem:  
 $10 \cdot 10 \cdot X = 50$

We can see that X must be  $\frac{1}{2}$  to solve this equation. Therefore, we can write a formula for the area of a triangle, where 10 represents both the base and the height. We can derive the equation:

$$A = \frac{1}{2}bh$$

This is just one example, more areas can be derived by your students.

- Using tangrams, we have shown different ways to calculate area.  
What do you notice about the area you get using each method?

You should notice that no matter the method used, each will yield the same value.

Application 1:

Have students dilate the original tangram, rotate, reflect and translate its pieces. Have them make a three-dimensional model and introduce volume. If students have access to technology, have them experiment and model the shapes in *Geometer's Sketchpad*.

Application 2:

Have students work with spatial relationships to see how many shapes they can construct. See if they can use two pieces to form a triangle or even if they can make all seven pieces form a triangle.

Next time you are asked to find area, you will be able to choose between the methods. It is always a good idea to check your answer by using a second.

Notice that tangrams can be applied to many grade levels and are a great way to visually demonstrate concepts.