

Making Calc I Click

We will develop a further understanding of the first derivative of the position function and velocity. This will involve using a Calculator Based Ranger(CBR) to make this hands on. This gets the students actively involved and has them discover for themselves that the velocity is the first derivative.

Standards:

NCTM:

- generalize patterns using explicitly defined and recursively defined functions;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;

NYS MST:

7. Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

Students will be able to:

- recognize, describe, extend, and create a wide variety of patterns.
- represent and describe mathematical relationships.
- use a variety of manipulative materials and technologies to explore patterns.
- interpret graphs.

Objectives:

Students should be able to:

- take a velocity functions and find certain "check" points, i.e. plugging in certain values in for x in seconds to find y in feet per second.
- take these velocities and use the relationships between them to know how and when they need to change their speeds.
- take these velocities they found, and some given check points to walk the position function
- understand why/how the first derivative will give you the velocity.

Instructional Protocol/Itinerary:

- Introduce the velocity and it's relation to the first derivative
- Read and explain hook.
- Explain the CBR and how it works.
- State the instructions for our activity(make sure students understand what the $-/+$ mean)
- Run through an example to show students, and then form two teams for them to do it on their own.
- Relate back to the hook and conclude that this is an activity to get students actively involved with the first derivative of the position function is the velocity function.

A Reverse Hook:

Vapid Velocity

This exercise is to be used at the beginning of a lesson to get students thinking about better ways to make connections to the velocity function.

Calculus I problem 1:

Suppose a ball is dropped from the upper observation deck of the CN Tower, and its position at time t is given by the function:

$$f(t) = \frac{1}{2}(t - 5)^2$$

Find the velocity of the ball after 3 seconds.

While any calculus student should know to take the derivative of the given position function and plug $t = 3$ into it, how many students would make a worthwhile connection to throwing a ball off the CN Tower? There must be a better way to model this position equation so that students can have a better understanding of what the first derivative tells us.

Calculus I problem 2:

If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height in meters t seconds later is given by:

$$y = 10t - 1.86t^2$$

Find the instantaneous velocity of the rock after 7 seconds.

Now this is not a bad question to work out; however what student has ever thrown a rock on Mars? An even better question is what connection could a student make between taking the derivative and plugging in 7 if they never have and will never be able to throw a rock on Mars? So we again think to ourselves that there must be a better way to model this, or any, position function for our students.

Using the CBL to create a plot on your TI-83/84:



PRESS ANY KEY

After selecting the CBL/ CBR application on your TI-83/84 this is the initial screen that you should see. This application is easily downloaded from <http://education.ti.com>, if not already present on your calculator. The first screen that appears after the title screen is an options menu for you to select the type of data you wish the CBR to collect. For our purposes we will be using the ranger feature of the application.

```
CBL/CBR APP:  
1: GAUGE  
2: DATA LOGGER  
3: RANGER  
4: QUIT
```

The ranger feature has many built in activities that students can do, however there are many innovative ideas that a teacher can come up with using the real time data collection ability of the CBR. To get to the real time data collection we choose option number one (Setup/ Sample) on the main menu.

```
MAIN MENU  
1: SETUP/SAMPLE  
2: SET DEFAULTS  
3: APPLICATIONS  
4: PLOT MENU  
5: TOOLS  
6: QUIT
```

Most of our examples will involve collecting distance data for 15 seconds from 1-15 feet.

Walk (maybe run) that Equation:

This activity is intended to give the students the derivative of a position function (the velocity equation) and a few points that the position function passes through. Then the students should be able to take the given information and walk the position function using the CBR.

How?

Well since the velocity function is given, the students will be able to know the velocity that they should be traveling at all times during the 15 seconds that they will be walking. Knowing this information, and the starting distance, a student should be able to walk a perfect graph. We can break any given velocity into two parts, its sign and its absolute value. The sign of a velocity at time t tells us the direction we should be walking. A negative sign means walking toward the CBR(decreasing distance) and a positive means away from the CBR(increasing distance). The absolute value will actually tell us our velocity. For example if we are given velocities of 2 and 4 ft/sec, and times 7 and 8 seconds respectively, we know we must double our speed between the 7th and 8th second of our walk.

Here is a Walk that Equation example (for a given position function y):

$$y' = x + 5$$

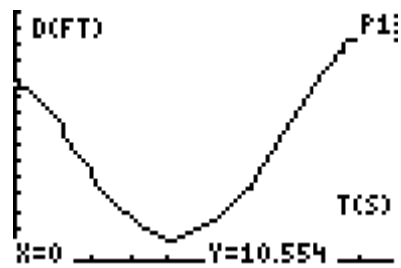
Check points from position function:

- At zero seconds be 12.5 ft from the CBR (starting point)
- At 5 seconds be 0 ft from the CBR

Now the students should make a table to site their times (in seconds) and velocities (in ft/sec):

<u>Time</u>	<u>velocity</u>
0	5
1	4
2	3
3	2
4	1
5	0
6	-1
7	-2
8	-3
9	-4
10	-5
11	-6
12	-7
13	-8
14	-9
15	-10

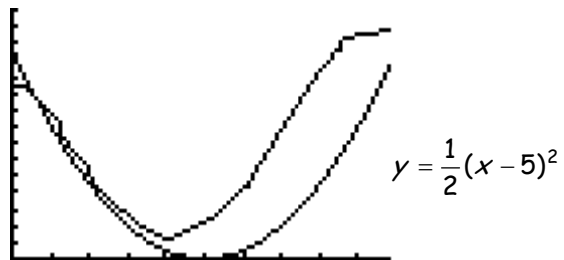
Using this data my attempt to walk that equation knowing only the velocities I should be traveling at looks as follows:



At this point the students could be told that the given derivative is derived from the equation

$$y = \frac{1}{2}(x - 5)^2$$

Superimposed over the equation we walked we get:



And students see that the second derivative really does give accurate velocities!

Closure -

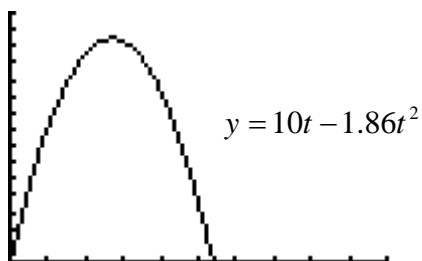
The Hook Revisited:

So we have seen an activity that allowed the students to physically walk the velocity that an equation told them too. Now we can return to the most boring velocity problem ever introduced in the hook and give it better context with our CBR knowledge.

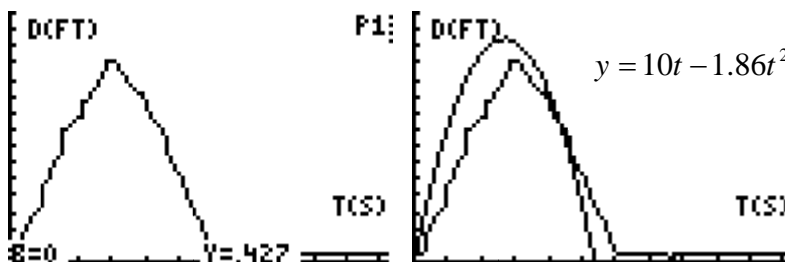
The equation that was given in the hook was:

$$y = 10t - 1.86t^2$$

Instead of this equation modeling the throwing of a rock on mars which has no connection to students we can use the CBR to be able to physically move out the velocities at different points of the graph. Just graphing this velocity on a calculator gives the second derivative very little meaning to the students:



However having a student try and walk the this equation out before we graph it on the calculator not only gives meaning to the different velocities but also gets the students physically moving along the curve of the function



The graph on the left is an attempt at modeling the curve through walking. However when the actual graph of the function is superimposed over the model created it is obvious the starting velocity isn't enough to match that of the function. TADA! An instant connection is made about velocity.

Name: _____

Date: _____

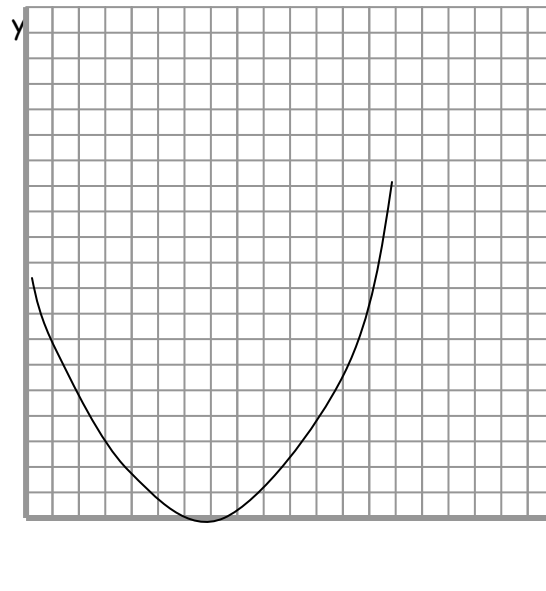
Calculus I
Velocity Functions

1)

a) Take your velocity function below and find the velocity at each of the times listed.

Velocity Function	Time(s)	Velocity(ft/s)
$y = x - 5$	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	11	
	12	
	13	
	14	
	15	

b) Once you've found your velocities, take your information over to the CBR and see if you can walk out the function using those speeds. Draw the graph as best you can in the grid below.



c) Compare the curve that you drew/walked to the given curve above. (ex. "At 4 seconds I should have walked faster.")

d) The curve given in the graph above does not represent velocity function. What do you think it represents?

2) Take the following position functions and use each to fill in the rest of the charts.

Position Function	Velocity Function	Time(s)	Position	Velocity
$y = x^2 - x + 2$		1		
		2		
		3		
		4		
		5		
		6		
		7		
		8		
		9		
		10		
		11		
		12		
		13		
		14		
		15		

Position Function	Velocity Function	Time(s)	Position	Velocity
$y = \frac{(x-8)^2}{8} + 1$		1		
		2		
		3		
		4		
		5		
		6		
		7		
		8		
		9		
		10		
		11		
		12		
		13		
		14		
		15		

Name: Answer Key

Calculus I
Velocity Functions

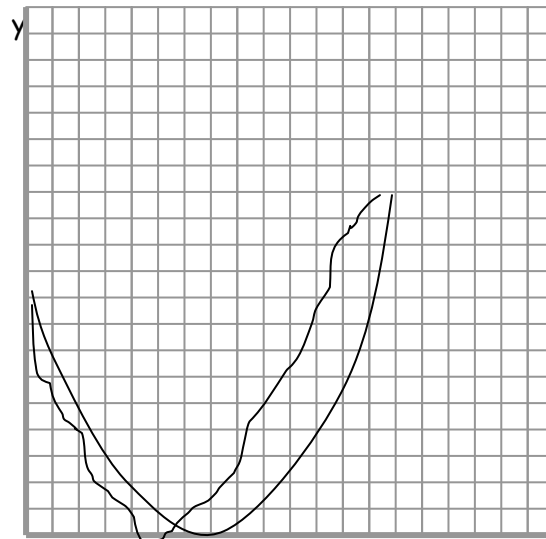
Date: _____

1)

a) Take your velocity function below and find the velocity at each of the times listed.

Velocity Function	Time(s)	Velocity(ft/s)
$y = x - 5$	1	-4
	2	-3
	3	-2
	4	-1
	5	0
	6	1
	7	2
	8	3
	9	4
	10	5
	11	6
	12	7
	13	8
	14	9
	15	10

b) Once you've found your velocities, take your information over to the CBR and see if you can walk out the function using those speeds. Draw the graph as best you can in the grid below.



x

c) Compare the curve that you drew/walked to the given curve above. (ex. "At 4 seconds I should have walked faster.") **I was pretty good with the graph on the way down, but I messed up by coming back up too soon.**

d) The curve given in the graph above does not represent velocity function. What do you think it represents?

The position function, because the velocity function is the derivative of the position function.

2) Take the following positions function and use each to fill in the rest of the chart.

Position Function	Velocity Function	Time(s)	Position	Velocity
$y = x^2 - x + 2$	$y' = 2x - 1$	1	2	1
		2	4	3
		3	8	5
		4	14	7
		5	22	9
		6	32	11
		7	44	13
		8	58	15
		9	74	17
		10	92	19
		11	112	21
		12	134	23
		13	158	25
		14	184	27
		15	212	29

Position Function	Velocity Function	Time(s)	Position	Velocity
$y = \frac{(x-8)^2}{8} + 1$	$y' = \frac{x-8}{4}$	1	7.125	-1.75
		2	5.5	-1.5
		3	4.125	-1.25
		4	3	-1
		5	2.135	-.75
		6	1.5	-.5
		7	1.125	-.25
		8	1	0
		9	1.125	.25
		10	1.5	.5
		11	2.125	.75
		12	3	1
		13	4.125	1.25
		14	5.5	1.5
		15	7.125	1.75