SUNY Fredonia
Department of Mathematical Sciences

MATH 210 – Mathematical Structures and Proof

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Catalog Description:

MATH 210 Mathematical Structures **A**

Careful study of the concepts and techniques often used in mathematics courses at the advanced undergraduate level. Topics include: logic, set theory, proof techniques, elementary number theory, mathematical induction, functions, and relations. Additional topics from abstract algebra, combinatorics, or countable vs. uncountable sets as time permits.
Prerequisite: MATH 121 or MATH 123.

4 hours credit

Rationale:

This course provides a careful study of the concepts and techniques needed to study mathematics at the advanced undergraduate level. It serves as a bridge from more concrete, procedure-driven courses, such as calculus, to the more abstract, proof- and application-oriented courses at the junior and senior level. The course emphasizes the development of the student's ability to generalize and abstract mathematical concepts – to read, speak, and write proper mathematics – including the ability to read, understand, and do proofs. As such, most universities now require a course such as MATH 210 for all mathematics majors.

The most recent curriculum recommendations from the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America, (CUPM Curriculum Guide 2004) stipulate that courses designed for mathematical sciences majors should ensure that students:

- Develop mathematical thinking and communication skills.
  - Progress from a procedural/computational understanding of mathematics to a broad understanding encompassing logical reasoning, generalization, abstraction, and formal proof;
  - Become skilled at conveying their mathematical knowledge in a variety of settings, both orally and in writing.
- Develop skill with a variety of technological tools.
  - All majors should have experiences with a variety of technological tools, such as computer algebra systems, visualization software, statistical packages, and computer programming languages.
- Provide a broad view of the mathematical sciences.
  - All majors should have significant experience working with ideas representing the breadth of the mathematical sciences. In particular, students should see a number of contrasting but complementary points of view: continuous and discrete; algebraic and geometric; deterministic and stochastic; theoretical and applied;

- Majors should understand that mathematics is an engaging field, rich in beauty, with powerful applications to other subjects, and contemporary open questions.

MATH 210 was designed with these recommendations in mind, and is also consistent with the following learning goals of our mathematics program:

- The ability to organize data and information and synthesize problem solutions using appropriate mathematical tools;
- The ability to formulate conjectures, find counterexamples, and state and prove theorems carefully;
- The facility to learn mathematics outside of the classroom through self-study or group-study, including the ability to refresh knowledge encountered in previous coursework;
- The ability to communicate mathematics effectively both orally and in writing;
- The facility to use technology to aid in the above.

Here, the second and fourth goals are particularly relevant for MATH 210.

In addition, a MATH 210 is particularly relevant for those students intending to teach mathematics, and it is a required course for each of the following majors:

- Early Childhood and Childhood Education with a Mathematics Concentration
- Mathematics/Middle Childhood Education
- Mathematics/Adolescence Education

In its recent report, *The Mathematical Preparation of Teachers*, the Conference Board of the Mathematical Sciences (CBMS) states that, to be well-prepared to teach the new *Standards*-based curricula, elementary, middle, and high school mathematics teachers need to develop a "deep understanding" of school mathematics concepts and procedures. Such knowledge will enable teachers to "assess their students' work, recognizing both the sources of student errors and their students' understanding of the mathematics being taught." It will also enable teachers to "appreciate and nurture the creative suggestions of talented students" and allow teachers to see the "links between different mathematical topics and make their students aware of them."

In its recommendations for elementary teacher preparation, the report goes on to note that teacher candidates "must believe that mathematics is about ideas that make sense, rather than a collection of motiveless rules" and that "among the obstacles to improved learning at the elementary level, not the least is that many teachers were convinced by their own schooling that mathematics is a succession of disparate facts, definitions, and computational procedures to be memorized piecemeal." Recognizing this, mathematics courses for prospective elementary teachers must begin by "helping teachers make meaning for the mathematical

objects under study – meaning that was often not present in their own [education] – and only then moving on to higher orders of generality and rigor." MATH 210 is designed to be in line with these precepts, as well as with the report's call for helping elementary teacher candidates develop a deep understanding of mathematics that "includes acquiring a rich network of concepts extending into the content of higher grades; a strong facility in making, following, and assessing mathematical argument; and a wide array of mathematical strategies."

As for prospective middle grades teachers, the CBMS report calls for courses that "will strengthen these prospective teachers' own knowledge of mathematics and broaden their understanding of mathematical connections between one educational level and the next." It indicates that "discrete mathematics can offer teachers an opportunity to explore in depth many of the topics they will teach."

Similarly, as concerns the preparation of high school mathematics teachers, the CBMS report notes the increasing importance of discrete mathematics in the curriculum, and the increasing use of technology to facilitate connections between discrete mathematics and the more traditional high school topics in algebra, analysis, and geometry. It recommends that, to be well-prepared to teach such curricula, prospective teachers need:

In the area of Algebra and Number Theory:

- Understanding of the properties of the natural, integer, rational, real, and complex number systems;
- Understanding of the ways that basic ideas of number theory and algebraic structures underlie rules for operations on expressions, equations, and inequalities;
- Understanding and skill in using algebra to model and reason about real-world situations;
- Ability to use algebraic reasoning effectively for problem solving and proof in number theory, geometry, discrete mathematics, and statistics.

In the area of Geometry and Trigonometry:

- Understanding of the nature of axiomatic reasoning and the role that it has played in the development of mathematics, and facility with proof.

In the area of Functions and Analysis:

- The ability to translate information from one representation (tables, graphs, or formulas) to another;
- The ability to use functions to solve problems in calculus, linear algebra, geometry, statistics, and discrete mathematics.

The National Council of Teachers of Mathematics, in its *Principles and Standards for School Mathematics*, has identified five content standards and five process standards that "specify the understanding, knowledge, and skills that students

should acquire from prekindergarten through grade 12." MATH 210 is directly relevant to the following NCTM standards:

- Number and Operations Standard
  - Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
  - Understand meanings of operations and how they relate to one another.
- Algebra Standard
  - Understand patterns, relations, and functions.
  - Represent and analyze mathematical situations and structures using algebraic symbols.
- Problem Solving Standard
  - Build new mathematical knowledge through problem solving.
  - Solve problems that arise in mathematics and in other contexts.
  - Apply and adapt a variety of appropriate strategies to solve problems.
  - Monitor and reflect on the process of mathematical problem solving.
- Reasoning and Proof Standard
  - Recognize reasoning and proof as fundamental aspects of mathematics.
  - Make and investigate mathematical conjectures.
  - Develop and evaluate mathematical arguments and proofs.
  - Select and use various types of reasoning and methods of proof.
- Communication Standard
  - Organize and consolidate mathematical thinking through communication.
  - Communicate mathematical thinking coherently and clearly to peers, teachers, and others.
  - Analyze and evaluate the mathematical thinking and strategies of others.
  - Use the language of mathematics to express mathematical ideas precisely.
- Connections Standard
  - Recognize and use connections among mathematical ideas.
  - Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

    - Recognize and apply mathematics in contexts outside of mathematics.
- Representation Standard
    - Create and use representations to organize, record, and communicate mathematical ideas.
    - Select, apply, and translate among mathematical representations to solve problems.

Textbook: *Mathematical Structures and Proof*, by H. Joseph Straight and Julia M. Wilson, or *A Spiral Workbook for Discrete Mathematics*, by Harris Kwong.

Objectives: Students who complete the course should:

- Be able to analyze mathematical statements and arguments using the tools of logic;
- Know when and how to apply the set operations of union, intersection, difference, complement, and product, as well as the power set operation;
- Know the basic strategies of proof, and be able to outline a direct proof, a proof by contrapositive, and a proof by contradiction;
- Be able to outline and know when to use a proof by induction;
- Know and be able to apply the division algorithm and the Euclidean algorithm;
- Know the fundamental theorem of arithmetic and be able to factor a positive integer into primes;
- Be able to determine whether a given function is one-to-one or onto;
- Be able to compose functions and find the inverse of a bijective function;
- Be familiar with the properties of functions under the operation of composition;
- Be able to determine whether a given relation on a set is reflexive, irreflexive, symmetric, antisymmetric, or transitive;
- Be able to determine whether a given relation on a set is an equivalence relation, and know and be able to apply the fundamental theorem of equivalence relations;
- Be able to do simple proofs, such as showing that two sets are equal, or proofs involving the concepts of divisibility, relatively prime integers, primes, modular arithmetic, functions, and/or relations.

Instructional Methods and Activities: Lecture/Discussion

Refer to individual instructor syllabus for additional details.

Evaluation and Grade Assignment: Typically will involve homework, short quizzes, exams, and a comprehensive final exam.

Refer to individual instructor syllabus for additional details.

Grading Scale:

Refer to individual instructor syllabus.

Bibliography:

1. Robert B. Ash, *A Primer of Abstract Mathematics*, Mathematical Association of America, 1998.
2. Edward B. Burger, *Extending the Frontiers of Mathematics: Inquiries into Proof and Argumentation*, Key College Publishing, 2007.
3. Gary Chartrand, Albert D. Polimeni, and Ping Zhang, *Mathematical Proofs: A Transition to Advanced Mathematics*, 2nd ed., Addison-Wesley, 2008.
4. Richard Johnsonbaugh, *Discrete Mathematics*, 5th ed., Prentice Hall, 2001.
5. Kenneth H. Rosen, *Discrete Mathematics and Its Applications*, 6th ed., McGraw-Hill, 2006.
6. Edward R. Scheinerman, *Mathematics: A Discrete Introduction*, 2nd ed., Thomson Brooks/Cole, 2006.
7. Diane Driscoll Schwartz, *Conjecture and Proof: An Introduction to Mathematical Thinking*, Saunders, 1997.

In addition, the following journals frequently contain articles related to discrete mathematics and accessible to undergraduates:

1. *The American Mathematical Monthly*, published by the Mathematical Association of America.
2. *The College Mathematics Journal*, Mathematical Association of America.
3. *Mathematics Magazine*, Mathematical Association of America.
4. *Mathematics Teacher*, National Council of Teachers of Mathematics.
5. *Mathematics Teaching in the Middle School*, National Council of Teachers of Mathematics.

Topics:

Logical connectives; logical equivalence; logical quantifiers; sets and set operations; proof techniques; mathematical induction; division algorithm; Euclidean algorithm; primes numbers and the fundamental theorem of arithmetic; modular arithmetic; one-to-one and onto functions; composition of functions; inverse functions; relations and their properties; equivalence relations; other topics as time permits.

Policies: Refer to individual instructor syllabus.