Outline for Mathematical Structures and Proof
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Jonathan Cox

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Relevant sections of the text are noted in parentheses after a topic.

I. Introduction to statements and proof

- A. Motivation
- B. Evidence and conjecture (3.1)
- C. The nature of proof
- D. Sentences
- E. Statements (3.1)
- F. Properties (3.1)
- G. Quantified statements (3.6)
- H. Existentially quantified statements
- I. Proving existentially quantified statements
- J. Universally quantified statements
- K. Proving universally quantified statements
- L. Negation (3.1)
- M. Disproving universally quantified statements
- N. A few comments about sets (should go earlier)
- O. Disproving existentially quantified statements
- P. Negating multiply quantified statements (3.6)

II. Deductive reasoning and connectives

- A. Preview of symbolic logic
- B. Arguments and validity
- C. Form and connective symbols
- D. Truth functions and truth tables
- E. Conjunction (3.2)
- F. Disjunction (3.2)
- G. Conditional implication (3.3)

III. Logical equivalence and direct proof

- A. Tautologies and contradictions (3.5)

B. Basic rules of inference (5.3)
    1. Modus ponens
    2. Hypothetical syllogism
    3. Disjunctive syllogism
C. The biconditional (3.4)
D. Equivalence (3.5)
E. Some important equivalences (3.5)
F. Proofs in symbolic logic
G. Conditionals involving properties
H. Direct proof (5.3)

IV. Basics of number theory

A. Divisibility (6.1, 6.4)
B. Axioms and The Principle of Well-Ordering (6.2)
C. The Division Algorithm (6.3)
D. The Euclidean Algorithm (6.5)
E. Modular arithmetic (6.8)

V. Sets

A. Set membership and set identity (4.1)
B. The empty set (4.1)
C. Cardinality (in brief) (4.1)
D. Subsets (4.2)
    1. Definition
    2. Proving $A \subseteq B$
    3. Proving equality of sets
    4. Power set
    5. Venn diagrams
E. Indirect proof: Proof by contradiction (5.4)
F. When to consider an indirect proof
G. Set operations
    1. Union (4.3)
    2. Intersection (4.3)

3. Complements (4.3)
4. Cartesian product (4.5)

VI. Induction

A. The Principle of Mathematical Induction (5.5, 5.6)

1. Dominoes
2. Well-ordered sets
3. PMI and PMI technique
4. Example(s)
5. Guidelines for format of an induction proof
6. Further notes on induction
7. Extended form of PMI technique

B. The Strong Principle of Mathematical Induction (5.7)

1. Strong PMI technique
2. Recursively defined sequences
3. The Fundamental Theorem of Arithmetic
4. All people are of the same sex (pitfalls of induction)

VII. Functions

A. Essentials (7.1)

1. Relationships between sets (relations)
2. Definitions of function and associated concepts
3. Showing that "a function is well-defined"
4. Functions as mappings
5. Rules of correspondence
6. Examples of important functions
7. Equality of functions

B. One-to-one, onto, and bijective functions (7.2)
    1. Onto functions
    2. Proving a function is onto
    3. 1-1 functions
    4. Proving a function is 1-1
    5. The Horizontal Line Test
    6. Bijections
    7. Proving a function is bijective
    8. Bijections and cardinality

C. Composition of functions (7.3)
    1. Definition (with equality of the intermediate sets)
    2. Computing formulas of composites
    3. Properties of the composition operation
    4. Inherited properties of composites
    5. Less restrictive condition under which a composition is defined

D. Inverse functions (7.3)
    1. Inverse relations
    2. Conditions for an inverse to be a function
    3. Properties of inverse functions
    4. Characterizations of the 1–1, onto, and bijective properties in terms of compositions
    5. Inverses in terms of composition

VIII. Equivalence relations

A. Relations
    1. Notation and terminology (7.4)
    2. Reflexive, symmetric, and transitive properties (7.5)

B. Equivalence relations and equivalence classes (7.6)
    1. Checking whether a relation is an equivalence relation
    2. Equivalence classes
    3. Congruence modulo n
    4. Properties of equivalence classes

IX. Combinatorics

A. Addition and multiplication principles (8.2)
B. The principle of inclusion-exclusion (8.2)
C. Permutations (8.3)
D. Combinations (8.3)
E. Further topics in combinatorics
F. Properties of the binomial coefficients (8.4)
G. Others as time permits