

Outline for Mathematical Structures and Proof

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Relevant sections of the Straight/Wilson text are noted in parentheses after a topic.

- I. Introduction to statements and proof
 - A. Motivation
 - B. Evidence and conjecture (1.1)
 - C. The nature of proof
 - D. Sentences
 - 1. Statements (1.1)
 - 2. Properties (1.1)
 - 3. A few comments about sets (1.1/2.5)
 - 4. The empty set (1.5)
 - 5. Definitions
 - 6. Some properties of integers (Even and odd, Divisibility)
 - E. Quantified statements (1.4)
 - 1. Introduction
 - 2. Existentially quantified statements
 - 3. Proving existentially quantified statements
 - 4. Universally quantified statements
 - 5. Proving universally quantified statements
 - 6. Negation and negating quantified statements
 - 7. Disproving universally quantified statements
 - 8. Disproving existentially quantified statements
 - 9. Proving and disproving multiply quantified statements

II. Deductive reasoning and connectives

- A. Preview of symbolic logic
- B. Arguments and validity (1.1)
- C. Form and symbols
- D. Truth functions and truth tables (1.2)
- E. Connectives (1.2)
 - 1. Conjunction
 - 2. Disjunction
 - 3. Conditional implication
 - 4. Biconditional implication
- F. Truth tables for more complex formulas (1.3)
- G. Determining validity of arguments using truth tables

III. Introduction to direct proof

- A. Conditionals involving properties
- B. Direct proof (1.6)

IV. Rules of inference and logical equivalence

- A. Tautologies and contradictions (1.3)
- B. Basic rules of inference (1.3)
 - 1. Modus ponens
 - 2. Hypothetical syllogism
 - 3. Disjunctive syllogism
 - 4. Minor rules of inference
 - 5. Recognizing rules of inference
- C. Logical equivalence (1.3)
- D. Some important equivalences (1.3)
- E. Proofs in symbolic logic
- F. Proofs involving conjunctions and disjunctions
 - 1. Conjunction in the consequent
 - 2. Proof by cases
 - 3. Disjunction in the consequent
 - 4. Proofs of statements involving biconditionals
 - 5. A first look at induction (1.7)

V. Sets

- A. Set membership and set identity (4.1)
- B. Subsets (1.1)
- C. Proving set containment (1.5)
- D. Proving equality of sets (1.5)
- E. Proper subsets (1.5)
- F. Universal sets (1.1)
- G. Venn diagrams (1.5)
- H. Cardinality (intuitive) (1.5)
 - I. Power set (1.5)
- J. Set operations (1.5)
 - 1. Union
 - 2. Intersection
 - 3. Complements
 - 4. Cartesian product
- K. More proofs involving sets

VI. Introduction to number theory and indirect proof

- A. Divisibility (2.2)
- B. A second look at induction: The Principle of Mathematical Induction (1.7)
 - 1. Dominoes
 - 2. Well-ordered sets
 - 3. PMI and PMI technique
 - 4. Axioms and The Principle of Well-Ordering (1.6/1.7/2.1)
 - 5. Example(s)
 - 6. Guidelines for format of an induction proof
- C. The Division Algorithm (2.2)
- D. Modular arithmetic (2.5)
- E. Indirect proof
 - 1. Overview
 - 2. Proving the contrapositive (1.6)
 - 3. Proof by contradiction (1.6)
 - 4. When to consider an indirect proof
- F. The Euclidean Algorithm (2.3)

VII. A third look at induction: Extending PMI

- A. Further notes on induction and examples (1.7)
- B. Extended form of PMI technique (1.7)
- C. The Strong Principle of Mathematical Induction
 - 1. Strong PMI technique (1.7)
 - 2. Recursively defined sequences (1.7)
 - 3. The Fundamental Theorem of Arithmetic (2.7)
 - 4. Detecting errors in induction proofs

VIII. Functions

- A. Essentials
 - 1. Relationships between sets (relations) (4.1)
 - 2. Definitions of function and associated concepts (3.1)
 - 3. Showing that “a function is well-defined” (3.1)
 - 4. Functions as mappings (3.1)
 - 5. Rules of correspondence (3.1)
 - 6. Examples of important functions (3.1)
 - 7. Equality of functions (3.3)
- B. One-to-one, onto, and bijective functions (3.2)
 - 1. Onto functions
 - 2. Proving a function is onto
 - 3. 1-1 functions
 - 4. Proving a function is 1-1
 - 5. The Horizontal Line Test
 - 6. Bijections
 - 7. Proving a function is bijective
 - 8. Bijections and cardinality

IX. Equivalence relations

- A. Relations
 - 1. Notation and terminology (4.1)
 - 2. Reflexive, symmetric, and transitive properties (4.2)
- B. Equivalence relations and equivalence classes (4.3)
 - 1. Checking whether a relation is an equivalence relation
 - 2. Equivalence classes
 - 3. Congruence modulo n
 - 4. Properties of equivalence classes

- X. Combinatorics
 - A. Addition and multiplication principles (5.2)
 - B. The principle of inclusion-exclusion (5.4)
 - C. Permutations (5.3)
 - D. Combinations (5.3)
 - E. Permutations with repetitions (5.4)
 - F. Combinations with repetitions (5.4)
 - G. Distributing balls into boxes (5.4)
 - H. Properties of the binomial coefficients (5.5)
- XI. Cardinalities of sets (if time permits) (Fall 2009 optional section)
 - A. Numerically equivalent sets
 - B. Denumerable sets
 - C. Uncountable sets
 - D. Comparing cardinalities of sets
 - E. The Schröder-Bernstein Theorem
- XII. More on functions (if time permits)
 - A. Composition of functions (3.3)
 1. Definition (with equality of the intermediate sets)
 2. Computing formulas of composites
 3. Properties of the composition operation
 4. Inherited properties of composites
 5. Less restrictive condition under which a composition is defined
 - B. Inverse functions (3.3)
 1. Inverse relations
 2. Conditions for an inverse to be a function
 3. Properties of inverse functions
 4. Characterizations of the 1–1, onto, and bijective properties in terms of compositions
 5. Inverses in terms of composition