SUNY Fredonia

Department of Mathematical Sciences

MATH 123 – University Calculus II

Course Coordinator: Dr. Meral Arnavut www.fredonia.edu/faculty/math/arnavut

Catalog Description:

MATH 123 University Calculus II A

Definite integrals, the fundamental theorem of calculus, techniques of integration, applications of the definite integral in the physical sciences and geometry, improper integrals, differential equations, sequences and series, and computational technology. Credit will not be given for both MATH 121 and MATH 123.

Prerequisite: MATH 122.

4 hours credit

Rationale:

This is the second course in a three-semester sequence devoted to the study of calculus. The primary focus of this course is integral calculus for functions of a single variable, which will often involve the inverse of problems presented in MATH 122. For example, techniques will be developed to determine a function from information pertaining to its rate of change, or more simply, determine the function whose derivative is specified. This process of antidifferentiation proves to be a very powerful tool used to compute the area between curves, volumes of solids, the distance along a curve, work, hydrostatic pressure, center of mass, consumer surplus, cardiac output, etc. Additionally, many concepts in integral calculus are used to solve problems that require derivatives to model *real world* phenomena such as moving bodies, heat transfer, compound interest, population growth, current flow, the spread of infectious diseases, etc. Hence, a strong understanding of this subject is required to fully comprehend the world around us. For this reason, it is not surprising that calculus is used in and serves as a gateway to many other scientific fields, such as physics, engineering, astronomy, biology, chemistry, the social sciences, etc. Additionally, the concepts and tools from calculus are considered fundamental to many other courses in mathematics.

The department has the following learning goals for its graduates:

A. The ability to organize data and information and synthesize problem solutions using appropriate mathematical tools;

B. The ability to formulate conjectures, find counterexamples, and state and prove theorems carefully;

C. The facility to learn mathematics outside of the classroom through self-study or group-study, including the ability to refresh knowledge encountered in previous coursework;

D. The ability to communicate mathematics effectively both orally and in writing;

E. The facility to use technology to aid in the above.

MATH 123 is designed to help students begin to meet all of these goals but particularly A.

The Mathematical Association of America, through its Committee on the Undergraduate Program in Mathematics (CUPM), has consistently recommended that courses in single and multivariate calculus be required of all math majors. Its most recent curriculum recommendations (CUPM Curriculum Guide 2004) stipulate that courses designed for mathematical sciences majors should ensure that students:

- Develop mathematical thinking and communication skills.
  - Progress from a procedural/computational understanding of mathematics to a broad understanding encompassing logical reasoning, generalization, abstraction, and formal proof;
  - Become skilled at conveying their mathematical knowledge in a variety of settings, both orally and in writing.
- Develop skill with a variety of technological tools.
  - All majors should have experiences with a variety of technological tools, such as computer algebra systems, visualization software, statistical packages, and computer programming languages.
- Provide a broad view of the mathematical sciences.
  - All majors should have significant experience working with ideas representing the breadth of the mathematical sciences. In particular, students should see a number of contrasting but complementary points of view: continuous and discrete; algebraic and geometric; deterministic and stochastic; theoretical and applied.
    Majors should understand that mathematics is an engaging field, rich in beauty, with powerful applications to other subjects, and contemporary open questions.

MATH 123 was designed with these recommendations in mind, and is also consistent with the following learning goals of our mathematics program:

- The ability to organize data and information and synthesize problem solutions using appropriate mathematical tools;
- The ability to formulate conjectures, find counterexamples, and state and prove theorems carefully;
- The facility to learn mathematics outside of the classroom through self-study or group-study, including the ability to refresh knowledge encountered in previous coursework;
- The ability to communicate mathematics effectively both orally and in writing;
- The facility to use technology to aid in the above.

Here, the first, third, and fifth goals are particularly relevant for MATH 123.

In addition, courses in calculus are extremely relevant for those students intending to teach mathematics. In particular, Early Childhood or Childhood Education majors with a Mathematics Concentration must take one of the following sequences: MATH 120/121, Survey of Calculus I and II, or MATH 122/123, University Calculus I and II. Hence, MATH 123 may be used to partially satisfy this requirement. The motivation for this requirement complements the recommendations provided by the Conference Board of the Mathematical Sciences (CBMS) in their recent report, *The Mathematical Education of Teachers*. Specifically, the report states that to be well-prepared to teach the new *Standards*-based curricula, elementary mathematics teachers need to understand the concepts and practices associated with algebra and functions including:

- Representing and justifying general arithmetic claims, using a variety of representations, algebraic notation among them; understanding different forms of argument and learning to devise deductive arguments.
- The power of algebraic notation: developing skill in using algebraic notation to represent calculation, express identities, and solve problems.
- Field axioms: recognizing commutativity, associativity, distributivity, identities, and inverses as properties of operations on a given domain; seeing computation algorithms as applications of particular axioms; appreciating that a small set of rules governs all of arithmetic.
- Function: being able to read and create graphs of functions, formulas (in closed and recursive forms), and tables; studying the characteristics of particular classes of functions on integers.

These skills are used repeatedly throughout MATH 123, especially, since functions are the primary object of analysis in calculus.

Additionally, MATH 123 targets many geometry and measurement competency areas for elementary teachers cited in the CBMS report, including:

- Visualization skills of two- and three-dimensional objects.
- Basic shapes, their properties and relationships among them, especially congruence and similarity.
- The process of measurement: understanding the idea of a unit, knowing the standard (English and metric) systems of units, and being able to convert measurements from one unit to another.
- Length, area, and volume formulas for basic shapes and understanding the independence of perimeter and area, of surface area and volume.

Skills in these areas are required to solve many word problems involving calculus applications.

Similarly, Childhood Education majors with a Middle School Extension must take one of the following sequences: MATH 120/121, Survey of Calculus I and II, or MATH 122/123, University Calculus I and II. Hence, MATH 123 may be used to partially satisfy this requirement. Majors in Mathematics/Middle Childhood

Education are required to take MATH 123. Rationale for these requirements is consistent with the recommendations in the CBMS report which states that, in the area of algebra and functions, prospective middle grades teachers should:

- Understand and be able to work with algebra as a symbolic language, as a problem-solving tool, as generalized arithmetic, as generalized quantitative reasoning, as a study of functions, relations, and variation, and as a way of modeling physical situations.
- Develop an understanding of variables and functions, especially of different equivalent relationships between variables.
- Understand linearity and how linear functions can illustrate proportional relationships.
- Recognize change patterns associated with linear, quadratic, and exponential functions.
- Demonstrate algebraic skills and be able to find a rationale for common algebraic procedures.

These skills are used repeatedly throughout MATH 123, especially, since functions are the primary object of analysis in the course.

Additionally, MATH 123 also targets, to a lesser extent, many geometry and measurement skills recommended for prospective middle school teachers cited in the CBMS report including:

- Identify common two- and three-dimensional shapes and list their basic characteristics and properties.
- Make conjectures about geometric shapes and then prove or disprove them.
- Demonstrate ability to visualize and solve problems involving two- and three-dimensional objects.
- Connect geometry to other mathematical topics, and to nature and art.
- Understand common forms of measurement.

Skills in these areas are required to solve word problems involving many calculus applications.

Since calculus is commonly taught in advanced placement form in many high schools across the country, Mathematics/Adolescence Education majors are required to complete MATH 123. The CBMS report specifies that in the area of functions and analysis, prospective high school mathematics teachers need a deep understanding of important classes of algebraic and transcendental functions and should be able to:

- Recognize patterns in data that are modeled well by each important class of functions.
- Identify functions with special properties such as periodicity.

- Recognize equations and formulas associated with each important class of functions and the way that parameters in these representations determine particular classes.
- Translate information from one representation (tables, graphs, or formulas) to another.
- Use functions to solve problems in calculus.
- Use calculator and computer technology effectively to study individual functions and classes of related functions.

As noted above, these skills are used repeatedly throughout MATH 123.

It is noted in the CBMS report, "Calculus and linear algebra courses provide an opportunity to give undergraduates extensive practice with algebraic manipulation. Making this an explicit goal for these courses helps to assure that future teachers have technical 'know how' in high school algebra." Naturally, this notion is certainly consistent with the goals of this course. Additionally, many of the modeling and application problems in MATH 123 address skills identified in the CBMS report for prospective high school mathematics teachers in the area of geometry and trigonometry including:

- Mastery of core concepts and principles of Euclidean geometry.
- Understanding trigonometry from a geometric perspective and skill in using trigonometry to solve problems.

The National Council of Teachers of Mathematics, in its *Principles and Standards for School Mathematics*, has identified five content standards and five process standards that "specify the understanding, knowledge, and skills that students should acquire from prekindergarten through grade 12." MATH 123 is directly relevant to the following NCTM standards and student expectations for Grades 9 – 12:

- Number and Operations Standard
  - Develop a deeper understanding of very large and very small numbers and various representations of them.
  - Judge the reasonableness of numerical computations and their results.
- Algebra Standard
  - Generalize patterns using explicitly defined and recursively defined functions.
  - Understand relations and functions and select, convert flexibly among, and use various representations for them.
  - Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
  - Understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, and use technology to perform such operations.

- Understand and compare properties of classes of functions including exponential, polynomial, rational, logarithmic, and periodic functions.
- Understand the meaning of equivalent forms of expressions, equations, inequalities, and relations.
- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency – mentally or with paper and pencil in simple cases and using technology in all cases.
- Use symbolic algebra to represent and explain mathematical relationships.
- Use a variety of symbolic representations, …, for functions and relations.
- Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
- Use symbolic expressions, …, to represent relationships arising from various contexts.
- Draw reasonable conclusions about a situation being modeled.
- Approximate and interpret rates of change from graphical and numerical data.

- Geometry Standard
  - Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them.
  - Use trigonometric relationships to determine lengths and angle measures.
  - Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

- Problem Solving Standard
  - Build new mathematical knowledge through problem solving.
  - Solve problems that arise in mathematics and in other contexts.
  - Apply and adapt a variety of appropriate strategies to solve problems.
  - Monitor and reflect on the process of mathematical problem solving.

- Reasoning and Proof Standard
  - Recognize reasoning and proof as fundamental aspects of mathematics.
  - Make and investigate mathematical conjectures.
  - Develop and evaluate mathematical arguments and proofs.

- Communication Standard
  - Organize and consolidate mathematical thinking through communication.

- Communicate mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

- Connections Standard
  - Recognize and use connections among mathematical ideas.
  - Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
  - Recognize and apply mathematics in contexts outside of mathematics

- Representation Standard
  - Create and use representations to organize, record, and communicate mathematical ideas.
  - Select, apply, and translate among mathematical representations to solve problems.

Textbook: *Calculus: Early Transcendentals*, 6th edition, by James Stewart, Thomson Brooks/Cole, 2008.

Calculator: A TI graphing calculator is strongly recommended. A TI-89 is preferred, but a TI-83 Plus or TI-84 is acceptable. The extent to which calculator use is necessary will depend on the instructor.

Online Homework: The online system *WebAssign* (https://www.webassign.net/) is used for some homework assignments. It helps students learn through practice and instant feedback.

Objectives: Students who complete the course should:

- recognize the definite integral as the limit of a Riemann sum, understand the relationship between definite integrals and areas and distances, be able to approximate definite integrals numerically and using area formulas, know how to approximate the definite integral of a function expressed in tabular form or graphically;
- realize the profound importance and utility of the Fundamental Theorem of Calculus for evaluating definite integrals, recognize differentiation and integration as inverse processes, be able to use the Fundamental Theorem of Calculus to evaluate definite integrals and differentiate functions defined in integral form such as the Fresnel function and appreciate their applicability and importance in other scientific fields;
- understand the concept of an antiderivative and know how to evaluate indefinite integrals to find general antiderivatives of functions, understand the difference between indefinite and definite integrals, recognize that the definite integral of the rate of change of a function represents the total amount

of change in the function over the interval specified by the limits of integration, know how to compute quantities such as distance traveled, change in concentrations of chemical reactions, mass of objects whose density is described by a function, change in production costs, etc.;

- be proficient at using the integration techniques of substitution, integration by parts, trigonometric substitution, and partial fractions when required, be able to evaluate and approximate indefinite and definite integrals using technology, recognize common functions whose antiderivatives cannot be represented in terms of elementary functions;
- know how to evaluate improper integrals, understand the relationship between improper integrals and limits, be able to test the convergence or divergence of indefinite integrals;
- be able to derive and evaluate integrals used for computing areas between two curves, volumes of solid objects by the slicing method, volumes of solids of revolution using the disk and shell methods, work, hydrostatic force, arc length, and surface area of revolution, understand the role of the differential in formulating integrals associated with measurable quantities;
- understand the concepts of, and differences between, sequences and series, be able to determine when a sequence is convergent or divergent; be able to recognize geometric series and telescoping sums and determine their convergence properties, understand the difference between conditional and absolute convergence, know how to use various tests (ratio test, root test, etc.) to establish the convergence properties of series, especially power series, be able to represent a function as a power series and know which functions have a power series representation, be proficient at computing the Taylor and MacLaurin series of functions and know how to approximate functions using Taylor polynomials, understand and appreciate the applications of Taylor polynomials;
- be comfortable solving problems requiring the use of integral calculus in many application areas, have a highly developed geometrical intuition of calculus concepts and a deep understanding and appreciation for its applications.

Instructional Methods and Activities: Lecture/Discussion

Refer to individual instructor syllabus for additional details.

Evaluation and Grade Assignment: Typically will involve homework, short quizzes, exams, and a comprehensive final exam.

Refer to individual instructor syllabus for additional details.

Grading Scale:

Refer to individual instructor syllabus.

Bibliography:

1. Howard Anton, Irl Bivens, and Stephen Davis, *Calculus: Early Transcendentals*, 8th ed., Wiley, 2005.
2. Monty J. Strauss, Gerald L. Bradley, and Karl J. Smith, *Calculus*, 3rd ed., Prentice Hall, 2002.
3. Richard St. Andre, *Study Guide for Stewart's Single Variable Calculus: Early Transcendentals, 6th ed.*, Brooks/Cole, 2007. (Amazon.com lists James Stewart as the author.)
4. Silvanus P. Thompson and Martin Gardner, *Calculus Made Easy*, St. Martin's Press, 1998.

In addition, the following journals frequently contain articles related to calculus and are accessible to undergraduates:

1. *The American Mathematical Monthly*, published by the Mathematical Association of America.
2. *The College Mathematics Journal*, Mathematical Association of America.
3. *Mathematics Magazine*, Mathematical Association of America.
4. *Mathematics Teacher*, National Council of Teachers of Mathematics.

The following sites on the internet may also prove to be useful:

1. http://www.math.arizona.edu/software/azmath.html

This web site contains links to software created by faculty at the University of Arizona for use in their mathematics classes. Among the useful links for this course is one that lets students test their readiness for calculus by means of a test of prerequisite material. Other links relating directly to calculus are readily available, and students are encouraged to visit this site and explore the material available.

2. http://www.karlscalculus.org/

Karl's Calculus Tutor is intended to provide you with clear, concrete, down-to-earth explanations of the material commonly encountered in calculus. Besides that, it will provide you with worked examples, and even more helpful than that, it will provide you with coached exercises in which you will solve hard problems by answering a series of easy questions.

3. http://www.math.uakron.edu/~dpstory/e-calculus.html

e-Calculus is a Calculus I tutorial written in TeX and converted to the Adobe Portable Document Format (PDF). Features include *verbose discussion of topics*, typeset quality mathematics, user interactivity in the form of multiple choice quizzes, in-line examples and exercises with complete solutions, and pop-up graphics.

4. http://www.sosmath.com/

S.O.S. MATHematics is a free resource for math review material from Algebra to Differential Equations. It is an excellent study site for high school, college students and adult learners.

5. http://www.stewartcalculus.com

This site is keyed to the textbook for this course and contains an algebra review with tutorial, additional topics, drill exercises, Problems Plus, History of Mathematics, downloadable versions of CalcLabs for *DERIVE* and TI graphing calculators.

Topics:

Definite integrals, the fundamental theorem of calculus, techniques of integration, applications of the definite integral in the physical sciences and geometry, improper integrals, differential equations, sequences and series, and computational technology.

Policies:

Refer to individual instructor syllabus.