1. Starting with a segment of unit length, explain how to construct a segment of length $\sqrt{6}$ using Euclidean tools. (Can you do it in two different ways?)

2. Suppose a Buffon needle experiment is performed with 1.2 cm between parallel lines and a needle of length 1 cm. Out of 426 total tosses, the needle lands on a line 230 times. To 6 places after the decimal, what estimate does this give for $\pi$?

3. In Problem 2, if the number of times the needle landed on the line were smaller, what effect would this have on the estimate for $\pi$?

4. Is it possible to cut up the interval $[3, 4]$ into pieces of exactly the same size such that one of the pieces has the point $\pi$ on its right edge? Why or why not?

5. Show that $\sqrt{7}$ is irrational.

6. Show that $\sqrt{2} + \sqrt{7}$ is irrational.

7. Let $a = .010010001000100001\ldots$ and $b = .101101110111011110\ldots$. Show that $a + b$ is rational. Yet it’s true that $a$ and $b$ are irrational. (Do you need to rethink your argument for Problem 6 in light of this?)

8. Show that $\log_{14} 9$ is irrational.

9. Using the series

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots,$$

how many terms have to be used to determine with certainty that the first place of $\pi$ after the decimal point must be 1?

10. Now use Machin’s formula $\pi/4 = 4 \arctan(1/5) - \arctan(1/239)$ with the first two terms in the series for each arctangent value. What estimate does this give for $\pi$? Compare this to the direct series approach of the previous problem.

11. **Solve using the Greek geometric method:** Apply to a segment $AB$ of length 13 a rectangle of area 40 so that the deficiency is a square. (To what quadratic equation does this correspond?) Geometrically, this involves finding a point $Q$ on $AB$ that determines the dimensions of the rectangle. Make sure to give the actual dimensions of the rectangle—these are the roots of the quadratic equation!

12. Verify that the method you used to determine point $Q$ in the previous problem works in general (i.e., gives a rectangle with the desired properties).

13. Determine what’s wrong with the following problem: Apply to a segment $AB$ of length 20 a rectangle of area 110 so that the deficiency is a square.

14. **Solve using the Greek geometric method:** Apply to a segment $AB$ of length 8 a rectangle of area 105 so that the excess is a square. (To what quadratic equation does this correspond?) Are the dimensions of the rectangle the roots of the quadratic equation this time? If not, what adjustment needs to be made?

More review problems might be provided next week. Let me know if you have specific requests.