Broad description of content: There will be questions from the reading assignments of April 20 and April 22, much like those that usually appear on the quizzes. Apart from these, the emphasis of the exam will be on 1) computations and proofs, often to be carried out using the (suspected) historical methods, and 2) explanations of why certain methods were used, why and how certain historical phenomena happened, why mathematics developed the way it did, etc. (as best as we can tell), rather than on details from the earlier readings.

Materials: During the exam you may use a calculator and two 3” × 5” notecard with notes written in your own handwriting. One notecard should only contain notes on the readings from 6.7-6.8 and 8.1-8.5, and the other can contain anything you want. This time I will collect your notecards when you turn in your exams. Scratch paper will be available for those who want to use it. You are also free to bring a ruler or compass, but these won’t strictly be necessary.

Test layout: I suspect the exam will be four pages.

Sections covered: 3.4-3.10, 4.1-4.8, 6.1-6.3 (and just the readings from 6.7-6.8 & 8.1-8.5)

Most important skills and concepts for this exam (in no particular order)

- Proofs in Greek geometric algebra (e.g., early propositions in Euclid’s *Elements*)
- Algebraic interpretation of Euclid’s geometric propositions; proving algebraic identities geometrically
- The Greeks’ three types of geometric problems that correspond to quadratic equations
- “Solving quadratic equations” by the Greek method
- Conic sections (definitions of Menaechmus and Apollonius, the three main types)
- Duplication, multisection, and quadrature problems
- The three great Greek construction problems
- Insolvability of the three construction problems using straightedge and compass (and connection to roots of polynomial equations)
- Solutions of the three construction problems using higher geometry, for example
  1. The two mean proportionals of Hippocrates
  2. Verging constructions and associated curves (especially conchoids)
  3. The spiral of Archimedes
  4. Solutions using conics
- Proving that a number is irrational (including π)
- Constructing a segment whose length is any square root
- Computations of the digits of π using
  1. the classical method
  2. infinite series
  3. the Buffon needle experiment
Other important concepts/topics
figurate numbers (triangular, square, pentagonal, etc.) and proofs of their properties
higher differences of sequences (e.g., second differences)
Proving that a number is happy (or sad)
Geometric proofs of the Pythagorean Theorem
Properties of Pythagorean triples
The Pythagorean theory of ratios
Incommensurable lengths and irrational numbers (including proofs)
The philosophy of Greek geometric algebra
Archimedes’ divisions of a sphere into segments
Constructing a sieve of Eratosthenes
Proving properties of Pythagorean triples (e.g., that a Pythagorean triple is or is not primitive)

Definitions: (quadrable) lune, Euclidean tools, straightedge, compass (Euclidean and modern),
incommensurable, spherical segment, conic (section), irrational number, transcendental number

A sheet with some review exercises will also be available.