Introduction:
This lesson is intended for Algebra students who have had exposure to the post March eighth grade material. Students should have a brief introduction to factoring polynomials and the definition of a polynomial. Students should be familiar with common factors, variables, and integers. For many, this particular lesson is difficult to understand. I will demonstrate several methods of factoring which work for all trinomials. More advanced concepts of factoring will be taught in Algebra 2/Trigonometry. I intend to touch upon the five models discussed by Lesh, Lamon, Gong, and Post. Using various instructional strategies will help learners understand the topic. This is intended to be a two-three day lesson.

Standards/Performance Indicators:
NYS Mathematics, Science, Technology Learning Standard 3:
☆ Problem Solving: Use multiple representations (A.PS.4)
☆ Reasoning and Proof: Extend results to more than one general case (A.RP.10)
☆ Communication: Communicate verbally and in writing design and explanation for steps used (A.CM.1), Use language of mathematics to express mathematical ideas precisely (A.CM.10-13)
☆ Connections: Recognize and make connections among multiple representations (A-CN.1-2)
☆ Representation: Create and use representations to organize, record and communicate mathematical ideas (A.R.1-3)
☆ Algebra: Factor algebraic expressions completely (A.A.20)
☆ Geometry: Find area of figures (A.G.1)

NCTM Standards Addressed:
☆ Algebra
☆ Geometry
☆ Problem Solving
☆ Reasoning and Proof
☆ Connections
☆ Communication
☆ Representation

Objectives:
At the conclusion of this lesson, students will:
☆ Factor trinomials using a method taught in class.
☆ Write trinomials as a sum of areas of rectangles.
☆ Represent trinomials and factors with pictures.
☆ Form real life applications to using trinomials.

Materials:
☆ Algebra tiles
☆ Graph paper
☆ Colored pencils
☆ Handouts

Instructional Protocol:
Begin lesson with a brief warm-up activity to review FOIL, GCF, and Common Factor Factoring. At this time, also briefly review trinomial factoring that was done in eighth grade.

Next, pass out algebra tiles or graph paper and colored pencils/markers.

Third, ask the students, review the basic unit of measure of the algebra tiles or draw standards on graph paper. This will be helpful during the next portion of the lesson.

Next, using the algebra tiles or graph paper and colored pencils/markers ask students to create rectangles that represent various trinomials listed in guided notes.

Using rectangles formed, label the sides and state the factors.

Begin Method 2, which is old fashioned trinomial factoring. This method is done without the use of manipulatives.

Applications for the Real World.

Helpful Hints:
- Most students struggle with factoring trinomials so spend time to master the skill.
- Give students support and things to look for.
- DO NOT teach every single method of factoring known to mankind all at once. It confuses students on what method to use. Teach a method that works every time and help them to discover what special cases exist along the way.

Evaluation/Closing Activities:
- As a closing activity, put FIVE different trinomials one the board/overhead/ELMO. Ask students to find the factors of each either by using algebra tiles/graph paper or algebraically.
- Extra Credit: Have students come up with a list of real world applications that were not discussed in class.
- Use warm-ups or quizzes to help students remember skills.
The Amazing Factoring Race

Factoring trinomials is an important lesson in learning Algebra. Since Eighth grade, you were introduced to the concept of factoring and polynomials. We are now going to build on what you already learned and make you feel comfortable with factoring. Before we jump into factoring, let's start with a little review.

Review of Polynomials:
Polynomial:
   A polynomial is a monomial or sum of monomials. For example: \(2x^3 + 3x^2 - 6x + 1\).
Trinomial:
   A sum of three monomials. For example: \(2x^2 + 3x + 1\).
Binomial:
   A sum of two monomials. For example: \(3x + 1\).
Monomial:
   A number, variable, or a product of a number and one or more variables. For example: \(3x\).

Polynomial Factoring:
Factoring a polynomial is the opposite process of multiplying polynomials. Recall that when we factor a number, we are looking for prime factors that multiply together to give the number. When we factor a polynomial, we are looking for simpler polynomials that can be multiplied together to give us the polynomial that we started with.

Removing Common Factors:
Factor out that common factor. You are doing is using the distributive law in reverse—you are sort of undistributing the factor by putting the factor on the outside of a set of parentheses and the remaining terms on the inside.

GCF:
Stands for the Greatest Common Factor. It is the largest factor that is a factor of both terms. It can be a number, variable or a combination of both numbers and variables.

Factoring the Floor
Julie is an interior decorator with a savvy sense of style. She likes to purchase boxes of tiles and create her own patterns for her customers. Unlike her sense of style, Julie has some difficulty when it comes to figuring out how many of each tile she needs. For her first design of the week, she knows that for each large tile sized, \(x^2\), she will need an area covering 5x by medium sized tiles and an area covering 6 by small tiles. What are the dimensions of the space she is covering if it can only be in the shape of a rectangle? By the end of the lesson you should be able to answer this question with confidence.

Using Manipulatives to Understand Trinomial Factoring:
Recall that manipulatives are often useful in relating concrete information to a concept that is more abstract. Since we spent a short amount of time looking at factoring before, let's try an alternate way of looking at it. First, using either Algebra Tiles or graph paper or the Edy tiles attached, familiarize yourself with each shape and it's area. For the graph paper, first draw, color and cut out the tiles or draw a key at the top of the page for reference and draw out all pictures. For the Edy tiles, cut out the tiles to be used. The large square is the \(x^2\) unit, the rectangle is the \(x\) unit, and the small square is the constant or in most cases 1.

To Use Tiles or graph paper or the Edy tiles:
Create a graph or diagram with tiles to represent the equation. The tiles represent the parts of the equation and can be manipulated to form rectangles. The large square represents the \(x^2\) term, the rectangle represents the \(x\) term, and the small square represents the constant term.

Determine how many of each sized tile you will need.
Arrange the tiles so that the large square and one of the small squares are diagonal to one another. If you have more than one small square arrange them so that they form a rectangle. If you have more than one \(x^2\), arrange them so they are next to one another forming a rectangle.
Next, fill in the remaining space with the rectangular shaped tiles.

To find the factors:
- Use the sides of the rectangle are adjacent to one another.
- Label each side based on how many $x$ units there are and how many constant units there are.
- For example, if the side has 3 rectangular units and 6 small squares, then it is $3x+6$.

Let’s do one together first. For example, let our trinomial be: $x^2 + 5x + 4$. Use the Tiles or graph paper or the Edy tiles to create rectangles representing each trinomial. Use the rectangles formed to find the factors of the trinomial.

First we need to decide how many of each sized tile we need. Since there is only one $x^2$ term, we will need one large square. There is a $5x$, which means we need $5$ rectangular blocks and there is a $4$, so we need for little squares.

Following the instructions listed above we need to put the large square and one of the small squares diagonal to one another.

Then we need to fill in the remaining space with the rectangular blocks to make a rectangle.
Finally we need to label the sides.

\[ x \quad 1 \quad 1 \quad 1 \quad 1 \]

\[ x \quad x \quad x \quad x \]

Our factors of \( x^2 + 5x + 4 \) are \((x+4)\) and \((x+1)\).

Practice:
Use the examples and the Tiles or graph paper or the Edy tiles to create rectangles representing each trinomial. Use the rectangles formed to find the factors of the trinomial.

\[ x^2 + 4x + 3 \]
\[ x^2 + 5x + 6 \]
\[ x^2 + 6x + 9 \]
\[ x^2 + 7x + 10 \]
\[ x^2 + 4x + 4 \]
\[ 2x^2 + 4x + 1 \]

Factoring like Mother used to do:
We will now look at an algebraic method of factoring. Similar to the manipulative method, some students may see this easier to understand first. Practice will make perfect with this particular topic so let’s get the show on the road. When we factor trinomials we are using the distributive property in reverse. In other words, we are taking a trinomial and putting it back the way it looked, two binomials in parentheses, before we multiplied them together or used FOIL.

First things first. Recall what we do when we multiply two binomials together or FOIL. For example, I have: \((x+2)(x+1)\). I will first multiply the First terms together, the \(x\) and \(x\), which is \(x^2\). Next I will multiply the Outside terms, the \(x\) and 1, and get \(x\). Third, I will multiply the Inside terms, the 2 and \(x\), and get 2\(x\). Last, I will multiply the Last terms, 2 and 1, which is 2. I will combine my like terms of \(x\) and 2\(x\), which is 3\(x\) and get the trinomial: \(x^2 + 3x + 2\).

When we factor a trinomial such as \(x^2 + 3x + 2\), we are putting it back into parentheses. If we look at the steps we took to multiply, we reverse the steps to factor. In other words, we do the following:
List all possible ways to get the coefficient of $x^2$ by multiplying two numbers.
- List all the possible ways of getting the constant term by multiplying two numbers.
- Try all possible combinations of these to see which ones give us the correct middle term.
- **Sign Rules:**
  - If the sign between the $x^2$ and the $x$ term is positive and the sign between the $x$ and the constant is positive, then the signs that will go in the parentheses are both positive.
  - If the sign between the $x^2$ and the $x$ term is positive and the sign between the $x$ and the constant is negative, then the signs that will go in the parentheses are opposites.
  - If the sign between the $x^2$ and the $x$ term is negative and the sign between the $x$ and the constant is positive, then the signs that will go in the parentheses are both negative.
  - If the sign between the $x^2$ and the $x$ term is negative and the sign between the $x$ and the constant is negative, then the signs that will go in the parentheses are opposites.
- Place all information in parentheses. Since we have an $x^2$ term, we always write one $x$ in the first term of each parenthesis.
- Check by multiplying the binomials together.

Let's look at two examples:

**Example 1:** Factor the following trinomial $x^2 + 5x + 4$

First consider the coefficient in front of the $x^2$ term. Since it is a 1, we know that the only numbers that multiply together to get a 1 is 1 and 1. Therefore, we can place an $x$ as the first term of each parenthesis:

$$(x) (x)$$

Next we need to consider signs. Since they are both positive, they will both be positive in parentheses.

$$(x+)(x+)$$

Third, we need to look at the numbers that give us four when we multiply them together. We are only look for two numbers here. Our choices are 2 and 2 or 1 and 4.

$$(x+1)(x+4) \text{ or } (x+2)(x+2)$$

If we check the first one, when we add the outside and inside terms together we get $5x$, which is the middle term of our trinomial. Therefore our answer is: $(x+1)(x+4)$.

**Example 2:** Factor the following trinomial $2x^2 + x - 3$

First consider the coefficient in front of the $x^2$ term. Since it is a 2, we know that the only numbers that multiply together to get a 2 is 2 and 1. Therefore, we can place a $2x$ in one parenthesis and an $x$ in the other:

$$(2x)(x)$$

Next we need to consider signs. Since they are opposite signs they will be opposite signs in parentheses so we will need to wait to put them in.

Third, we need to look at the numbers that give us negative three when we multiply them together. We are only look for two numbers here. Our choices are -1 and 3 and 3 and -1. We use the negative in front of the number as a subtraction sign in parentheses because it would be rather confusing to have two signs in the same parentheses.
\[(2x+1)(x-3) \text{ or } (2x-1)(x+3) \text{ or } (2x+3)(x-1) \text{ or } (2x-3)(x+1)\]

Last we need to check to see set of binomials gives us \(x\) as a middle term when multiplied together. If we look at the first one we get \(-5x\) as a middle term, which is too small. If we look at the second one we get \(5x\) as a middle term, which is too big. If we look at the third one we get \(x\) as a middle term, which is exactly what we were looking for. Therefore our answer is: \((2x+3)(x-1)\).

Practice:

\[a^2 + 2a + 1\]
\[x^2 + 5x + 6\]
\[x^2 - x - 6\]
\[y^2 + 13y + 40\]
\[3z^2 + 10z + 8\]
\[m^2 + 8m - 16\]

Real World Uses, to answer the question “Why do we need to learn this?”

The question nearly always arises, “Why do we need to learn this?” In fact, at times you may ponder the same thing. Trinomials are hardly something that you visualize as you walk down the street, or do you? Actually, looking back at the first method of factoring trinomials, we can visualize trinomials as we walk down the street. Remember, we can find factors using rectangles and if we know what the variable, usually \(x\) is, we can actually find the dimensions of the rectangle. So if for example, you are walking down a street that has cement block or brick layers, you can see a trinomial in action.

Julie, our interior designer from the first paragraph sees trinomials all the time. Although she may struggle with them at times, she uses them to create patterns for tiling. She might expand her line to add wallpaper and drapery since she is starting to see how trinomials work.

Trinomials can be used to figure out prices and numbers of tickets sold for a concert. This is more complex than the trinomials we were working with today, but I bet you will agree that there is a certain price that tickets can be sold for that would maximize the amount of money earned. The same holds true for any type of merchandise, houses, etc. In other words, mathematics drives the economy. Without trinomials, companies would play a guessing game to figure out how to make a profit, which would either benefit us greatly or hurt us, depending on how well they could guess.

I challenge you to find other real world examples that trinomials are needed for.