Picture This: Picture Functions
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Introduction:
This lesson is attended for students in grades 9-12, although handouts and guidelines may easily be manipulated to be used in lower grade levels. This lesson introduces patterns and their relationship to arithmetic and geometry, as well as an introduction to using tables as representing patterns and writing the function notation for a pattern. This lesson is intended to be used as a group activity, but can be modified to be an individual lesson.

Standards/Performance Indicators:
NYS Mathematics, Science, Technology Learning Standard 3:
☆ Problem Solving: Observe and Explain Patterns (A.PS.3), Use multiple representations (A.PS.4)
☆ Reasoning and Proof: Extend results to more than one general case (A.RP.10)
☆ Communication: Communicate verbally and in writing design and explanation for steps used (A.CM.1), Use language of mathematics to express mathematical ideas precisely (A.CM.10-13)
☆ Connections: Recognize and make connections among multiple representations (A.CN.1-2)
☆ Representation: Create and use representations to organize, record and communicate mathematical ideas (A.R.1-3)
☆ Algebra: Recognize, use, and represent algebraically patterns, relations, and functions (A.A.29-31)

NCTM Standards Addressed:
☆ Algebra
☆ Geometry
☆ Problem Solving
☆ Reasoning and Proof
☆ Connections
☆ Communication
☆ Representation

Objectives:
At the conclusion of this lesson, students will:
☆ Recognize patterns by verbal description or visual representation.
☆ Describe next term in pattern or sequence using verbal and/or visual representation.
☆ Predict other terms in pattern/sequence using verbal and/or visual representation.
☆ Represent pattern in tabular form using table.
☆ Create at least one unique pattern using manipulatives listed below.
☆ AS A BONUS: State the function notation for the patterns given.

Materials:
☆ Blocks or pattern blocks
☆ Pennies
☆ Graph paper
☆ Colored pencils
☆ Handouts
Instructional Protocol:

☆ Give students 5-10 minutes to "investigate" the patterns given on Student Handout Page 1. This can be given to each student or used as a small group activity. During this time, tell students to keep answers to themselves or within their group. Students should write something about the patterns that describe what is happening.

☆ Next, discuss with the class some of the different answers they may have formulated. Allow students to ask questions and give a little wait time in between patterns to allow for processing.

☆ Third, ask the students, either alone or in groups, to find the next term in the sequence. At this point they may use whatever methods necessary to find the pattern. Some students may find it helpful at this point to use some of the materials listed above for a more "hands on" approach to problem solving. This portion should take about 10-15 minutes.

☆ Next, incorporate the use of a table to help find the pattern. Demonstrate one with the students (an example is given in the handouts) and allow them to try one or two of the patterns on Student Handout Page 2 using the table. Allow them time to write in words what is actually happening.

☆ Helpful Hints:
  ○ Introduction to triangular, square, and pentagonal numbers can fit well here. It is not necessary to understand the lesson, but is helpful for finding the function.
  ○ Give students hints, such as look for constants or finding a unit that is common throughout.

Evaluation/Closing Activities:

☆ As a closing activity, ask students to pick one pattern from Additional Picture Functions and do the following:
  ○ Describe the pattern in writing. Be specific.
  ○ Predict the number of units the 6th and 10th terms will have.
  ○ Predict how the shape will look for each successive term.

☆ Another closing activity: As students to write their own pattern using geometric figures.

☆ Extra Credit Worthy: Write the functions for the patterns given.
Ok, so patterns are everywhere, but how are they related to math??? Well, fortunately for you, there are many ways to represent patterns in math, not only pictorially, but also algebraically, and verbally. We have been looking at patterns since, well since the day we were born but we didn’t know what they were then. So how exactly do we find a pattern?

First Things First:

☆ Patterns are easiest to see in pictorial form for many students, so we will start there.
☆ Begin by looking at the first term (picture) in the pattern; notice its structure (shape).
☆ Next, compare it to the next term in the pattern. What is the same? What is different?
☆ Continue with the third term in the pattern. What changes occur from the first to the second term? What about from the second to the third?
☆ Continue this process until you see the pattern that is formed.
☆ Another method is to look at the pattern as a whole. Try to establish a relationship between parts of the shape.

Now that we have the basic concept of how to find a pattern, look at the one below:

What are some of the things you notice? What is the structure or shape of the first term? What happens in the second term and the third term?

Sample Explanations:

☆ The first unit is a trapezoid and each term has one more trapezoid than the one before it.
☆ There is one less triangle in each term than there is a trapezoid.
☆ You could also look at the number of edges and determine a similar relationship.
☆ Determining the number of edges compared to the number of vertices.

Using Student Handout Page 1, complete the exercises. For each pattern given, describe what is going on. In words, provide details about what the pattern is and ways that it can be formed. The more specific you are now, the easier it will be later.

Next, let’s compare our answers. How do your answers compare to your neighbor’s answers? Are they the same? Are they different?

Using the same patterns you just described, find the next term in each of the patterns. You may use any tools available to you in the classroom to help you figure them out. Time is ticking... If you finish early, try to find the next term in each pattern. If you are successful you will have added two new terms to each pattern.

Time is up!! Now that you have familiarized yourself with patterns and can build more terms in a pattern, let’s look at things another way. If I give you a table, can you describe your pattern in it? Let’s use the example from above to help us.

Given the following pattern:

We need to write it in a table. To do so, follow these steps:
First, create a table.
Label it with something that describes what we are comparing. For this example the rows might be labeled number of trapezoids and term.
Fill in the table
Look for the pattern by looking at each row separately and then comparing the rows.
Make some predictions about how many units are in the 12th term.

<table>
<thead>
<tr>
<th>Term</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td># of trapezoids</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the example above, the first term is a single trapezoid. The second term has two trapezoids, and the third term has three trapezoids. If we continued on in this fashion we would have 4 trapezoids for the 4th term and 5 trapezoids for the 5th term.

Tables are especially useful when the pattern is not easily recognized from a picture. Now that you know how to use a table with a pattern, complete the examples on Student Handout Page 2. It would be good to include written description of what is going on as well as your tables.

Ok, do you have the hang of it??? Now, the very last thing to do is to write the rule or function for the pattern. For every pattern, there is a rule that can be written in numeric form instead of pictorial form. We can use the relationships discovered in the table and earlier observations to figure out what the pattern is.

Some basics on finding the function:
☆ The term is always part of the function. In other words, we usually let the term be represented by n and it is always multiplied by or added to something. Don’t forget we can ALWAYS multiply by 1.
☆ If there is a constant term that we are adding each time then we will be multiplying that constant by n.
☆ We can use any real number operations, although sticking to some of the basics (adding, subtracting, multiplying, and dividing) are usually easiest.
☆ Don’t forget to use some common sense once in a while.

Using the table above, we see that the total number of units, in this case trapezoids, that we have is equal to the term. Therefore, our function would be: \( A_n = n \).

Why is finding the function or rule useful? First, it helps to find terms that are not successive. For example, say I wanted to find the 32nd term. Are you going to draw all of them out? I hope not. The function allows us to substitute whatever term we trying to find in for n. For the example here I would replace n with 32 and simplify.

In more advanced mathematics, finding the function to a pattern simplifies the process of drawing everything out. In the end, you will be using arithmetic to find the pattern or the total number of units for that specific term.
Handouts:
Using the instructions provided in class: describe each pattern in words. After writing out the descriptions try to find the next term in each. If you finish early, find the next term beyond the one you just found.
Complete the following exercises using a table to describe the pattern. Along with your table, write a brief explanation in words.
Additional Picture Functions:
Function Representations to All Examples:

\[3n + 1\]
\[n(n + 1)\]
\[4n\]
\[2n + 1\]
\[6n(n - 1) + 1\]
\[3n(n - 1) + 1\]
\[\frac{3n^2 + 3n + 2}{2}\]
\[n(2n - 1)\]
\[2n\]
\[8n - 5\]
\[n^2\]
\[12n - 3\]
\[3n + 1\]
\[\frac{n(n + 1)}{2}\]
\[\frac{2n^2 - n}{2}\]
\[2n - 1\]
\[3n + 1\]
\[n^2 + 1\]