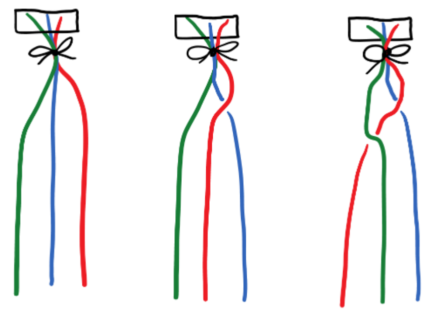
Braids

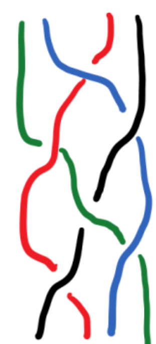
Leader Checklist

- ☐ Read through the module.
- ☐ Gather supplies: Yarn, scissors, tape



- Create a simple braid using three strands. Cut three pieces of yarn the same length (about 20 inches works well). Use a small piece of yarn to tie them together at one end, and then tape that end to your desk top to secure it. Bring the right strand over the center strand. Then bring the left strand over what is now the center strand. Repeat these two basic moves to create a braid of your desired length.

- Notice that each basic move consists of passing a strand either over or under the strand to its right! We call these basic moves *elementary braids*. By combining elementary braids, we can create braids that are complex as we like.

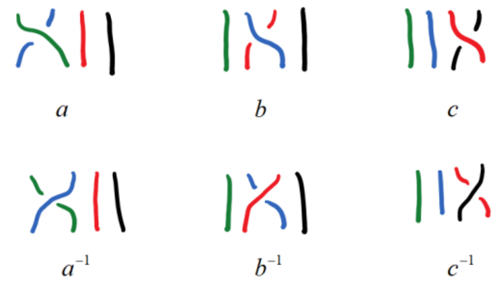


- Create a simple braid using four strands, as shown in this *braid diagram* to the right. If you like how it looks, you can turn it into a book mark or a decorative zipper pull!

- Consider the elementary braid that consists of passing the first strand over the second strand. Let's call this *a*. If we then pass the first strand *under* the second strand, then this undoes the crossing, and the strands hang free. In a sense, that second move is the *inverse* of the first move, since it undoes it. We'll call it a^{-1}.

a $\quad$ a^{-1} $\quad$ $aa^{-1}=1$

- The *unbraid* is the braid where all the strands just hang free, with no crossings at all! If we represent the unbraid with 1, then the equation $aa^{-1}=1$ captures the fact that the braid *a* followed by its inverse a^{-1} is really the unbraid in disguise.



- If we have four strands altogether, let's use the letters *a*, *b*, and *c* to refer to the elementary braids that consist of passing each of the first three strands over the strand to its right, and a^{-1}, b^{-1}, and c^{-1} for their inverses. The four-strand braid that you made earlier can be represented by the string of letters $bc^{-1}a^{1}bc^{-1}a^{-1}$. Can you see why?

- Try drawing a braid diagram for the braid $abccba$, or braiding it with yarn. If there are other four-strand braids that you know how to make, try representing them with braid diagrams or with strings of letters.

- If we only have three stands altogether, then we have two elementary braids, *a* and *b*, and their inverses. Try representing the simple braid you did at the beginning with a string of letters.

- If we have five strands of yarn, then we need four elementary braids *a*, *b*, *c*, and *d*, and their inverses. Try representing the following strings with braid diagrams, or actually braiding them with yarn: $abcddcba$, $ad^{-1}bc^{-1}$.

- Let's go back to four strands, with elementary braids *a*, *b*, and *c* and their inverses, as pictured above. See if you can explain why the following equation is true: $ac = ca$. Is $ab = ba$ true?

- Go big! Search online for instructions for braids with 5, 6, or more strands. See if you can braid them with yarn, or represent them with braid diagrams or strings of letters.

- Follow-up:
  - Let's let *n* represent the number of strands we're working with. The set of all possible braids on *n* strands is denoted by B_n, and it is generated by $n-1$ elementary braids and their inverses. Every braid in B_n can be represented as a sequence of those elementary braids and their inverses. Read about *braid groups* online or in a book.
  - Use this QR code to watch a video with beautiful animations illustrating the ideas we've learned about – elementary braids, inverses, and the structure of the braid group.