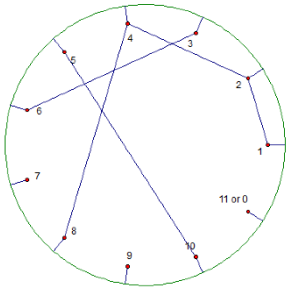
Mod String Art

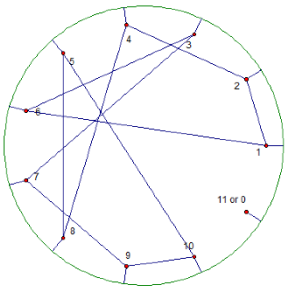
Leader Checklist

- ☐ Read through the module.
- ☐ Make string art disks in advance: Make copies of the template circles (see end of module). Glue circles to thin cardboard (poster board or cereal boxes) and cut them out. Use scissors to cut a notch at each line.
- ☐ Gather additional supplies: Medium thickness string ("crochet thread" or "upholstery thread" are good choices), scissors

- We're going to create craft projects by using string to join up pairs of notches on a circle. For each project, there are two numbers that matter: The total number of notches (n) and the multiplier (k).

- Here's an example where $n = 11$ and $k = 2$.
  - If a notch is labeled with the number x, then we join it to the notch with label $2x$. So we join up 1 and 2, 2 and 4, 3 and 6, etc. (We're doubling.)
  - In the first diagram, we've gotten as far as joining 5 and 10. But we're not done! We need to join up 6 and 12, but we don't have a notch labeled 12. So here's the cool idea: We reuse old notches! The next notch after 11 is labeled with a 1, but we'll also use it for 12, since 12 is one more than 11. We'll use the notch labeled with a 2 for 13, and the notch labeled with a 3 for 14 – you get the picture. When we're done, it looks like the second diagram.





- See if you can complete a diagram for $n = 11$ and $k = 3$.
  - Look for an empty diagram at the end of the module with 11 notches, and draw edges joining each number with *three* times that number: 1 and 3, 2 and 6, etc.
  - This time, you might have to reuse each notch more than twice.
  - For example, when you multiply 8 times 3, you get 24. Which notch acts like 24? You've gone all the way around the circle twice (11 got relabeled with a 22), plus two more notches to get up to 24, so use the 2. That is, join 8 to 2.

- Here's another way to think about all this:
  - In this setting (a circle with $n=11$ notches), 2 and 13 and 24 are "equivalent" numbers. They aren't *equal* numbers, but they're *equivalent* when $n=11$. They're each 2 more than a multiple of 11, and in fact *any* number that is 2 more than a multiple of 11 is equivalent to these. Can you find some more numbers that are equivalent to 2?
  - All the numbers that are 1 more than a multiple of 11 (1, 22, 34, 45, ...) are considered equivalent to each other.
  - Can you come up with more classes of numbers that are equivalent to each other when $n=11$?
  - Let's incorporate 0 and negative numbers, too! Since 0 is a multiple of 11, then it goes in the class with the other multiples of 11, namely $\{...,-22,-11,0,11,22,33,44,55,\ldots\}$.
  - What about -17? It's 5 more than a multiple of 11, since $-17=11\times(-2)+5$, so it goes with all the other numbers that are 5 more than a multiple of 11.
  - I bet you could now take *any* whole number and put it in the correct equivalence class!

- See if you can complete diagrams for $n=11$ and other choices of multiplier *k*.

- Ready to switch things up? Try a different value for *n*! You'll need to rethink your equivalence classes now. For example, when $n=20$, here are some equivalence classes:

  $\{...,-40,-20,0,20,40,60,80,\ldots\}$

  $\{...,-39,-19,1,21,41,61,81,\ldots\}$

  $\{...,-38,-18,2,22,42,62,82,\ldots\}$

  Can you find others?

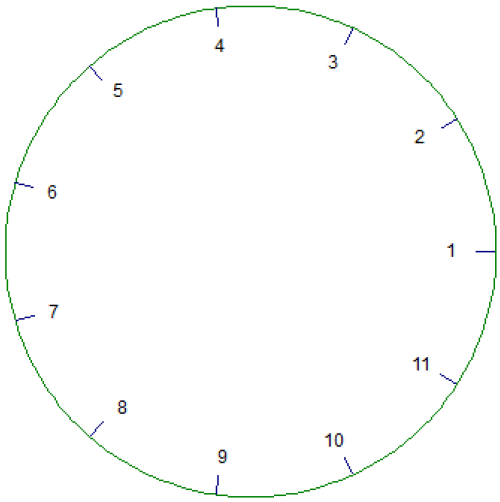
- Complete a diagram for $n=20$ and $k=3$. Look for an empty diagram at the end of the module that has 20 notches.

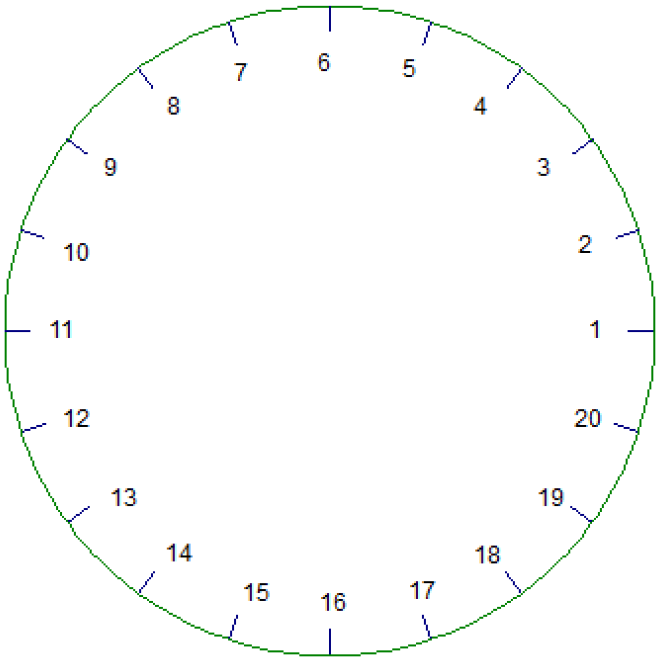
- Go big! See if you can complete a diagram for $n=80$, for some choice of multiplier *k*.

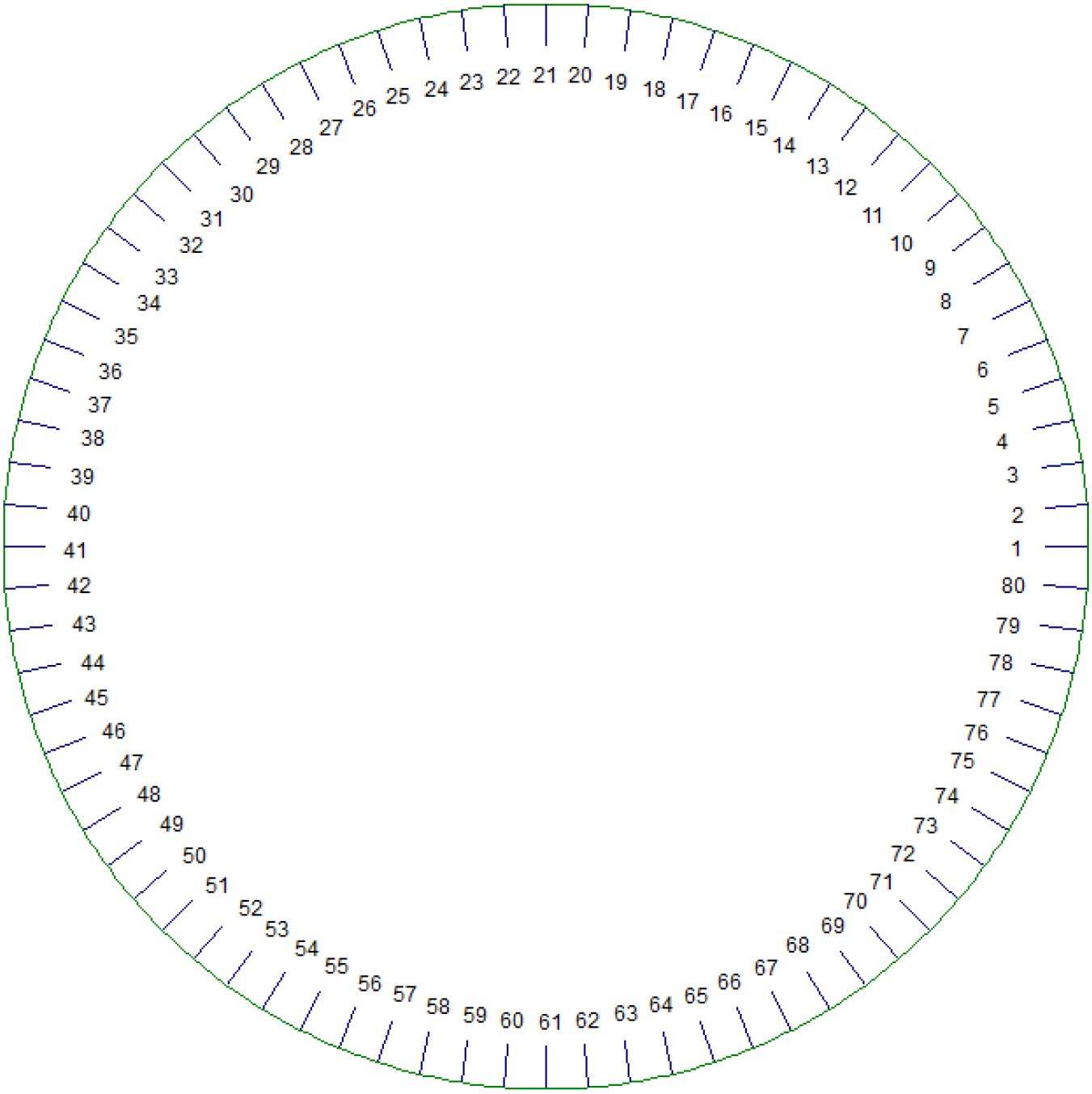
- Do you want to make a beautiful craft project? Glue the circle to thin cardboard, cut it out, and make small cuts for all the notches. Then use string to connect the notches.
  - Tie a knot in the string, and run the string through the first notch (labeled 1), with the knot on the back side of the disk. Pull the string until the knot keeps it from pulling all the way through. Right now, most of the string is on the front side of the disk.
  - On the front side of the disk, run the string from 1 to the notch you're joining it to (whatever *k* is), and pull the string through that notch so that it's now on the back of the disk again. Pull it taut so that the string makes a nice edge joining the notches on the front side.
  - Now keep going: Come up at notch 2 and go down at notch $k2$. Come up at notch 3, and go down at notch $k3$. Keep going, joining up *x* to kx for all numbers *x*, all the way up to $x=79$. (What would happen if you kept going after that?)
  - Tip: Choose a small number for *k*, such as 2, 3, 4, or 5.
  - Or don't – choose a bigger number if you like! The arithmetic gets harder, but if you're up for a challenge, go for it!

- If you run out of string, just start a new one and tie it to the old one on the back side of the disk.
- If you want to waste less string on the back of the disk, when you go to join x to kx, you can come up at either x or kx, whichever is closer. Just remember that your next edge will be from $x+1$ to $k(x+1)$.
- What do you think of your completed projects? Do you see some symmetry? How many petals are there, and how does this relate to your choice of multiplier k?

- Follow-up:
  - If you want to learn more, use the QR code to watch a video that has some great information.
  - You can also look online or in math books for information on *modular arithmetic* and curves like the *cardioid* and the *nephroid*.
  - Suppose it's currently 8:00. What time will it be in 1706 hours? Notice that $1706 = 12 \times 142 + 2$, so it's 2 more than a multiple of 12. There are a lot of real world applications where we're more interested in the *remainder* in a division problem than we are in the *quotient*.



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