

## Leader Checklist

Read through the module. (Yup, that's all!)

- Play a few rounds of Rock Paper Scissors.
- Let's change up the rules a little: Rock beats both paper and scissors, scissors beats paper, but paper doesn't beat anyone. Try playing this game. It's not as good a game now, is it? What went wrong?
- Now, are you ready for Rock Paper Scissors Lizard Spock? The rules are simple - use the QR code to watch a clip from The Big Bang Theory!
- Here's a good way to capture the rules of our various games:


balanced

not balanced

balanced
- In this kind of game, the objects that you can choose among are called weapons, and a balanced game is one in which each weapon beats as many other weapons as it loses to.
- If we represent the rules of the game in a diagram, like the ones above, then if it's a balanced game, each vertex has the same number of edges coming into it as it has going out of it.
- Is it possible to have a balanced game with four weapons? Can you make a good game called Rock Paper Scissors Lizard?
- What about six weapons? Seven? For which values of $n$ is there a balanced game with $n$ weapons?
- Follow-up:
- The diagrams above are examples of graphs, from an area of mathematics called Graph Theory. A graph is a collection of vertices, some of which are joined by edges. Look online or in a math book for more information on graph theory.
- These particular graphs are completely connected graphs, meaning that every pair of vertices is joined by exactly one edge. Furthermore, they are directed graphs, meaning that every edge has an arrow on it that gives it a sense of direction. Notice that in the graphs above, you can follow the arrows on a journey through the graph that travels along every edge exactly once, and returns to its starting point. Such a journey is called an Euler circuit, named after the great mathematician Leonhard Euler, who is credited with some of the earliest results in graph theory. Read about Euler paths and circuits. Not all graphs have them! Which ones do?
- Think about this: If you have a completely connected graph with $n$ vertices, and if you can find an Euler circuit in the graph, then that gives you a way to put arrows on all the edges in a way that is guaranteed to be balanced (at each vertex, as many arrows go in as there are arrows going out). So it comes down to this: For which $n$ does the completely connected graph have an Euler circuit?

